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Penrose limits, deformed pp-waves and the string duals of $\mathcal{N} = 1$ large-N gauge theory

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Abstract: A certain conformally invariant $\mathcal{N} = 1$ supersymmetric SU(N) gauge theory has a description as an infra-red fixed point obtained by deforming the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory by giving a mass to one of its $\mathcal{N} = 1$ chiral multiplets. We study the Penrose limit of the supergravity dual of the large-N limit of this $\mathcal{N} = 1$ gauge theory. The limit gives a pp-wave with R-R five-form flux and both R-R and NS-NS three-form flux. We discover that this new solution preserves twenty supercharges and that, in the light-cone gauge, string theory on this background is exactly solvable. Correspondingly, this latter is the stringy dual of a particular large charge limit of the large-N gauge theory. We are able to identify which operators in the field theory survive the limit to form the string's ground state and some of the spacetime excitations. The full string model, which we exhibit, contains a family of non-trivial predictions for the properties of the gauge theory operators which survive the limit.

Keywords: Superstrings and Heterotic Strings, Penrose limit and pp-wave background, AdS/CFT and dS/CFT Correspondence.

Contents

1. Introduction and conclusions

The AdS/CFT correspondence [1, 2, 3] has given us a concrete example of how to realise the old expectation [4] that large-N SU(N) gauge theory may be written as a theory of strings. Unfortunately, most of the direct computations on the Anti-de Sitter (AdS) side of the correspondence are not inherently stringy. This is largely due to the fact that the string theory background contains a non-trivial Ramond-Ramond (R-R) five-form flux, and the quantisation of string theory in such backgrounds is a task not yet fully understood in generality. Instead, much use has been made of the supergravity limit for reliable computations.

Nevertheless, there are many computations which have provided useful links between the supergravity truncation and the full superstring theory. Examples are those involving and, indeed, *requiring* explicit recourse to the extended nature of the branes which underlie the geometry. Branes expanding *via* the dielectric mechanism [5] into giant gravitons [6] are needed to understand the "stringy exclusion principle" [7]. It has been partially shown [8] how the same mechanism underlies the process whereby the type-IIB theory produces the rich family of vacua corresponding to the $\mathcal{N} = 4$ Yang-Mills theory broken to $\mathcal{N} = 1$ by adding masses to all three chiral multiplets [9]. Meanwhile the enhançon mechanism [10] has been shown [11, 12] to be crucial in supplementing singular supergravity duals [13, 14] in order to yield the correct description of the $\mathcal{N} = 2$ gauge theory vacua. Finally, while supergravity techniques are sufficient to calculate the wavefunctions associated with various glueball states — see, for example, refs. [15, 16] — the soft high energy behaviour of string scattering amplitudes is an essential ingredient in matching expected behavior for scattering these states in the dual gauge theory [17].

While these examples and many others in the same spirit are interesting scenarios in which to observe the truly stringy nature of the AdS/CFT correspondence at work, they are still indirect, and help only in characterising the available vacua of the theory. They provide only a small window of understanding of many dynamical issues; for this, one would need to consider aspects of the full string theory in the AdS background.

Important progress has recently been made however, at least in the case of a much simpler background. It has been shown [18] that the type-IIB supergravity theory has another maximally supersymmetric solution besides Minkowski space and $AdS_5 \times S^5$. This solution is a "pp-wave" with a null R-R five-form flux switched on. As anticipated in ref. [18], it seems natural that such a highly symmetric solution should have a role in the AdS/CFT discussion. Furthermore, it was shown [19] that string theory in this background is exactly solvable in the light-cone gauge. In fact, it was discovered [20, 21, 22] that there is a direct connection to be made: pp-waves can be obtained from any supergravity solution *via* the so-called "Penrose limit" [23, 24], and in fact the maximally supersymmetric wave is just such a limit of the $AdS_5 \times S^5$ geometry.

Correspondingly, one expects that there is a limit of the dual gauge theory which can be identified as the dual of type-IIB string theory on the resulting maximally supersymmetric pp-wave. Since the full type-IIB string theory in this background is solvable, one might also hope to finally be able to make direct and highly stringy statements about the gauge theory. These expectations have been borne out by the direct identification [21] of the set of operators which survive the limit, and furnish the string ground state and the spectrum of excitations. The relevant limit is to consider the sector charged under a $\mathrm{U}(1)_R$ subgroup of the full $\mathrm{SO}(6)_R$ R-symmetry of the theory, and take the charge J to be extremely large, growing as $N^{1/2}$. Sensible gauge theory results are obtained if at the same time g^2_{YM} is kept small and fixed and hence the 't Hooft coupling $\lambda = g^2_{\mathrm{YM}}N$ is sent to infinity. Since the 't Hooft coupling corresponds to the curvature scale in the string dual, which is $L = (g^2_{\mathrm{YM}}N)^{1/4}$ in units of the string coupling, this gauge theory limit fits nicely with the Penrose limit on the geometry, which also (as we will review) takes $L \to \infty$. We should now compare stringy results in the pp-wave background with gauge theory results in this limit. For example, the structure of the string hamiltonian alone makes a highly non-trivial prediction for the behaviour of the anomalous dimensions of a particular set of non- (but near-) BPS operators in the theory at large 't Hooft coupling:

$$(\Delta - J)_n = \sqrt{1 + \frac{n^2 g^2_{\mathrm{YM}} N}{J^2}} = 1 + \frac{n^2 g^2_{\mathrm{YM}} N}{2J^2} + \cdots . \tag{1.1}$$

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Here, Δ is the dimension of the operator and n labels an excitation level in the string theory on the one hand and, on the other, a particular type of operator in the dual gauge theory made by constructing ordered "words" of strings of field insertions under the gauge trace (see ref. [21]). The above prediction has been checked to leading order [21] in an appropriate new expansion parameter, $\lambda' = g_{\rm YM}^2 N/J^2$.

The hope is that the example above (and others in that spirit) will teach us new lessons about the AdS/CFT correspondence and about other more general gravity/gauge theory correspondences. The aim of this paper is to understand the meaning of this new facet of the correspondence in the context of a particularly interesting and useful class of backgrounds. These are the backgrounds which represent [25, 26] the "Holographic Renormalisation Group (RG) Flow" from the $\mathcal{N} = 4$ Yang-Mills theory to other gauge theories of interest, by a controlled deformation of the $AdS_5 \times S^5$ background. We have used what is considered (by some) as one of the cleanest and most instructive examples of such geometries, the flow to an infra-red (IR) $\mathcal{N} = 1$ supersymmetric fixed point, in which part of the dual's geometry is again conformal to AdS_5. While we have obtained some interesting results, and surmount a number of technical obstructions, we do not yet have a satisfactory understanding of all of our results, for reasons which will become clear later.

So for the main part of our presentation, we shall be concerned with a study of the endpoint of the flow, showing that the Penrose limit of the supergravity solution is again (not surprisingly) a pp-wave, but it is significantly different from previous examples which have been related to known gauge theories: in addition to the R-R five-form flux, it has R-R and Neveu-Schwarz-Neveu-Schwarz (NS-NS) three-form flux. It is therefore not maximally supersymmetric as in other recent examples [27, 28, 29] dual to large charge $\mathcal{N} = 1$ large-N gauge theories, but instead preserves twenty supercharges, possessing some of the unusual number of extra or "supernumerary" supersymmetries [30, 31, 32] beyond the standard half that pp-waves have generically.

Such a strange number of supersymmetries, as we predict here for a four-dimensional gauge theory is not immediately implausible. Recall that we have two special features: four-dimensional conformal invariance, and a special large charge limit which treats a particular direction in R-symmetry space as special. One achieves 32 supercharges in the usual context by first having the naïvely maximal sixteen, and then observing that the closure with the conformal algebra doubles each of the four supersymmetries (each equivalent to a single $\mathcal{N} = 1$ supersymmetry in four dimensions). It is conceivable that since we have picked a special direction out in R-symmetry space, there is a scaling limit on the superalgebra which can ensure that only one of the four supercharges gets doubled by demanding closure with the conformal algebra, giving the twenty we find here.[1]

Another direct prediction of our new model comes from deriving the hamiltonian of the string theory and comparing it to gauge theory in an analogous manner to the formula

[1] This is so far a conjecture as to how our prediction from the Penrose limit is realised, and it probably amounts to an Inonu-Wigner contraction of the full four-dimensional superconformal algebra. It is comforting to note that such limits are known to yield interesting subalgebras of the superconformal group in two dimensions, (see for example refs. [33] and references therein), and so it is worth exploring in four dimensions. For a recent discussion which may be relevant, see ref. [34].

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in eq. (1.1). While we have a sector of the operator spectrum which produces a formula like the above (with an appropriate redefinition of J to what we shall call $\hat{J}$; see later), we have a much more intriguing formula from another sector of the theory:

$$\left(\Delta - \hat{J}\right)_n^{\pm} = \sqrt{\frac{5}{2} + \frac{cn^2 g_{\rm YM}^2 N}{\hat{J}^2} \pm \frac{1}{2}\sqrt{9 + 12\frac{cn^2 g_{\rm YM}^2 N}{\hat{J}^2}}}\,, \tag{1.2}$$

where the constant c is given by the ratio of central charges of the fixed points of the gauge theory [46, 13]:

$$c = \frac{c_{\rm IR}}{c_{\rm UV}} = \frac{27}{32}\,.$$

Not only is the double square root an amusing challenge for the gauge theory side, observe further the form of the leading terms in the expansion:

$$\begin{aligned} \left(\Delta - \hat{J}\right)_n^{+} &= 2 + \frac{cn^2 g_{\rm YM}^2 N}{2\hat{J}^2} + \cdots\,, \\ \left(\Delta - \hat{J}\right)_n^{-} &= 1 + \frac{1}{6}\left(\frac{cn^2 g_{\rm YM}^2 N}{\hat{J}^2}\right)^2 + \cdots\,. \end{aligned} \tag{1.3}$$

We see that the latter sector contains a contribution which appears at second order in a λ' expansion, there being an exact cancellation at linear order. This either corresponds to a new class of diagrams on the gauge theory side, or a new exact cancellation in the usual diagrams. Since our exact string theory model is predicted to be dual to a known gauge theory, it will be a very interesting task to verify this behaviour.

The paper is organised as follows. We start our detailed discussion in section 2, where we consider the addition of a three-form to the supersymmetric pp-waves of type-IIB supergravity [18]. The resulting solutions cannot be maximally supersymmetric, but they are always at least one-half supersymmetric. Moreover, solutions preserving other exotic fractions can be constructed with relative ease. Although we do not undertake an exhaustive classification of all possibilities here, we present the specific solution which preserves 20 of the 32 supersymmetries; this seems to be the largest number that can be preserved in this case.

In section 3, we consider a Penrose limit of the ten dimensional fixed point geometry presented by Pilch and Warner [35], focusing on a null geodesic with large angular momentum in one of the directions corresponding to part of the moduli space of the gauge theory. The resulting solution includes a non-trivial three-form, and we find precisely the 5/8 supersymmetric pp-wave of section 2.

Our next step is to analyse the string spectrum in this background, which we do in section 4. String theory in this background is considerably more complicated than in the maximally supersymmetric case, studied in refs. [19, 36]. Moreover, although the effect of the three-form on the string spectrum is similar to cases considered in refs. [21, 37, 38], there are important differences. Since it is very interesting to have (potentially) the complete string theory dual of a sector of a four-dimensional gauge theory away from maximal supersymmetry, we remark on a number of features which may have some significance for the dual gauge theory.

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In section 5, we turn our attention to the study of the lowest lying sectors of the string theory and their comparison to a particular large charge limit of the $\mathcal{N} = 1$ superconformal gauge theory, which we of course conjecture to be dual to type-IIB string theory in this background.

While we have not fully understood the physical interpretation (in terms of a dual gauge theory) of our results for the entire flow geometry [39] representing the RG flow from the $\mathcal{N} = 4$ Yang-Mills theory to the IR fixed point, we present its Penrose limit in section 6. The resulting pp-wave is one-half supersymmetric at any point along the flow. Taking the Penrose limit thus enhances supersymmetry from 1/8 to 1/2 at any point along the flow geometry, and from 1/4 to 5/8 at the IR fixed point.

One of the technicalities to be faced in this RG flow context is the fact that such geometries are written naturally in Poincaré coordinates, while the Penrose limits of recent interest are mainly performed on AdS spaces written in terms of global coordinates. This leads to the interpretational difficulties alluded to above, since a simple trajectory in global coordinates representing a state (or class of states) in the field theory translates into a trajectory which probes a wide range of energy scales in the field theory. It is puzzling to interpret this cleanly in the dual field theory.

For completeness, and to summarise some of our conventions, we give an exposition of the ten-dimensional Pilch-Warner geometry [35, 39] in appendix A. In appendix B, we present a brief discussion of an alternate set of geodesics ($\theta = \pi/2$) and the associated Penrose limit of the IR geometry. Appendix C discusses a potential instability of superstring theory in the pp-wave backgrounds presented in section 2.1, and finally appendix D reviews the Penrose limit of $AdS_5 \times S^5$ in Poincaré coordinates [22, 40].

Note added in preparation: the results reported here were presented by C.V.J. at the workshop on Strings and Branes, held at KIAS, Korea, on May 29th of this year. While subsequently preparing this manuscript for publication, refs. [63, 64] appeared, which have some overlap with our results.

2. Adding three-forms to the supersymmetric IIB pp-wave

2.1 The generic solution

Since we want to retain the possibility of being able to exactly quantise superstring theory propagating on our pp-waves, we will only consider backgrounds with constant dilaton and axion. Without loss of generality we can take both to vanish. In that case, the bosonic sector of type-IIB supergravity contains the metric, an R-R four-form potential, C_4, with self-dual field strength, and NS-NS and R-R two-form potentials, B_2 and C_2, respectively. Combining these latter as $A_2 = B_2 + iC_2$, with field strength $G_3 = \mathrm{d}A_2 = H_3 + iF_3$, the self-dual five-form is given by[2]

$$F_5 = \star F_5 = \mathrm{d}C_4 - \frac{1}{8}\,\mathrm{Im}\,(A_2 \wedge G_3^*)\,, \tag{2.1}$$

where $*$ denotes complex conjugation and $\star$ denotes ten-dimensional Hodge duality. The equations of motion are then [41]

$$\begin{aligned} R_{ab} &= \frac{1}{6} F_{acdef} F_b{}^{cdef} + \frac{1}{8}\left(G_{acd} G_b^{*cd} + G_{acd}^* G_b{}^{cd} - \frac{1}{6} g_{ab} G_{cde} G^{*cde}\right), \\ \mathrm{d} \star G_3 &= 4i F_5 \wedge G_3\,, \qquad G_{abc} G^{abc} = 0\,, \\ \mathrm{d} \star F_5 &= \mathrm{d}F_5 = -\frac{1}{8}\,\mathrm{Im}\,(G_3 \wedge G_3^*)\,, \end{aligned} \tag{2.2}$$

and the remaining Bianchi identity is $\mathrm{d}G_3 = 0$. The fermionic sector of the theory consists of a dilatino, λ, and gravitino, ψ_a, with opposite chirality. Taking $\Gamma_{11} = \Gamma_0\Gamma_1 \ldots \Gamma_9$, we have

$$\Gamma_{11}\lambda = -\lambda\,, \qquad \Gamma_{11}\psi_a = \psi_a\,. \tag{2.3}$$

The supersymmetry transformations are given in terms of a single complex chiral spinor, ε, of the same positive chirality as the gravitino, and which can be written in terms of two sixteen-component Majorana-Weyl spinors: $\varepsilon = \theta^1 + i\theta^2$. For purely bosonic backgrounds one has [41]

$$\begin{aligned} \delta\lambda &= \frac{1}{24} G_{abc}\Gamma^{abc}\varepsilon\,, \\ \delta\psi_a &= D_a\varepsilon - \Omega_a\varepsilon - \Lambda_a\varepsilon^*\,, \end{aligned} \tag{2.4}$$

where we have defined

$$\Omega_a = -\frac{i}{480} F_{b_1\ldots b_5}\Gamma^{b_1\ldots b_5}\Gamma_a\,, \qquad \Lambda_a = \frac{1}{96}\left(G_{bcd}\Gamma_a{}^{bcd} - 9G_{abc}\Gamma^{bc}\right), \tag{2.5}$$

for later use.

The one-half supersymmetric pp-wave solution of the type-IIB theory presented in ref. [18] is

$$\begin{aligned} \mathrm{d}s^2 &= 2\mathrm{d}u\mathrm{d}v + H(u,x)\mathrm{d}u^2 + \mathrm{d}s^2\left(\mathbb{E}^8\right), \\ F_5 &= (1+\star)\mathrm{d}u \wedge \omega_4\,, \end{aligned} \tag{2.6}$$

where for each u, the four-form $\omega_4(u,x)$ is closed and co-closed (it can depend on u in an arbitrary way). The equations of motion relate H and ω_4 as

$$\nabla^2 H = -\frac{2}{3}\omega_4^2 \equiv -\frac{2}{3}\omega_{ijkl}\omega^{ijkl}\,, \tag{2.7}$$

[2]We take the field eqs. (2.2) and supersymmetry transformations (2.4) of type-IIB supergravity from ref. [41]. Our conventions are different, however, in that we use a "mostly plus" signature, but still have that $\{\Gamma_a, \Gamma_b\} = 2g_{ab}$. To write the results of ref. [41] in our conventions, we thus send $g_{ab} \to -g_{ab}$ and $\Gamma_a \to i\Gamma_a$, so that all our gamma matrices are real.

where $x^i, i = 1, \ldots, 8$, are the coordinates, and ∇^2 the laplacian, on $\mathbb{E}^8$. Taking $H = A_{ij}x^i x^j$, with A_{ij} a constant symmetric matrix, gives a Cahen-Wallach space [42]. With $\omega_4 = \mu\epsilon(\mathbb{E}^4)$ a constant multiple of the volume form on one of the transverse $\mathbb{E}^4$s, there is a unique choice[3] of $A_{ij} = -\mu^2\delta_{ij}$ for which the solution is maximally supersymmetric [18]. Switching on further constant components of ω_4 gives rise to pp-waves preserving between 16 and 32 supersymmetries [30, 31, 32], and some of these more general pp-waves further arise as Penrose limits of various intersecting brane solutions [22, 31, 43].

It is straightforward to switch on a non-trivial three-form, but one will then lose *maximal* supersymmetry. Since the Ricci scalar for the metric (2.6) vanishes, we have $G_{abc}G^{*abc} = 0$. Combining this with the equation of motion $G_{abc}G^{abc} = 0$, we must have that both F_3 and H_3 are independently null, in which case the three-form can be written as

$$G_3 = \mathrm{d}u \wedge \xi_2 \,, \tag{2.8}$$

for some complex two-form $\xi_2 = \alpha_2(u,x) + i\beta_2(u,x)$. Then, since $F_5 \wedge G_3 = G_3 \wedge G_3^* = 0$, all of the the eqs. (2.2) will be satisfied if, for each u, both ω_4 and ξ_2 are closed and co-closed, and if

$$\nabla^2 H = -\frac{2}{3}\omega_4^2 - \frac{1}{2}|\xi_2|^2 \,, \tag{2.9}$$

where $|\xi_2|^2 = \xi_{ij}\xi^{*ij}$.

One again finds that this generic solution preserves one-half of the 32 supersymmetries, those which satisfy $\Gamma^u\varepsilon = 0$ in the usual fashion. Other exotic fractions are possible, however, and although we leave a systematic study for future work, we will show here how to construct a solution which preserves 20 supersymmetries.

2.2 A 5/8 supersymmetric wave

We now specialize to the following:

$$\begin{aligned} \mathrm{d}s^2 &= 2\mathrm{d}u\mathrm{d}v + A_{ij}x^i x^j \mathrm{d}u^2 + \mathrm{d}s^2\left(\mathbb{E}^8\right), \\ F_5 &= \mu\ (1+\star)\mathrm{d}u \wedge \epsilon\left(\mathbb{E}^4\right), \\ G_3 &= \zeta\ \mathrm{d}u \wedge \mathrm{d}z^1 \wedge \mathrm{d}z^2 \,, \end{aligned} \tag{2.10}$$

where $z^1 = x^1 + ix^2$, $z^2 = x^3 + ix^4$ are complex coordinates on one of the transverse $\mathbb{E}^4$s, μ and ζ are real and complex constants, respectively, and the equation of motion (2.9) demands that

$$\mathrm{tr}\, A = -8\mu^2 - 2|\zeta|^2 \,. \tag{2.11}$$

As usual, we make use of the off-diagonal orthonormal basis

$$e^- = \mathrm{d}u \,, \qquad e^+ = \mathrm{d}v + \frac{1}{2}A_{ij}x^i x^j \mathrm{d}u \,, \qquad e^i = \mathrm{d}x^i \,, \tag{2.12}$$

with metric $\eta_{+-} = 1$ and $\eta_{ij} = \delta_{ij}$. The only non-trivial component of the spin connection is then

$$\omega_{-i} = \omega^{+i} = A_{ij}x^j \mathrm{d}u \,. \tag{2.13}$$

[3]Up to the differences in conventions between our work and that of ref. [18].

Consider, first, the dilatino variation

$$\delta\lambda = \frac{1}{8}\,\not{\xi}\,\Gamma_{+}\varepsilon = 0\,, \tag{2.14}$$

where we have defined $\not{\xi} = \xi_{ij}\Gamma_{ij}$. Any Killing spinors must thus satisfy

$$(1 - \Gamma_0\Gamma_9)\,(1 + i\Gamma_1\Gamma_2)\,(1 + i\Gamma_3\Gamma_4)\,\varepsilon = 0\,, \tag{2.15}$$

where we take the pp-wave to be moving in the x^9 direction. This equation has 28 independent solutions, characterised by the eigenvalues of the boost operator, $\Gamma_0\Gamma_9$, and rotation operators, $i\Gamma_1\Gamma_2$ and $i\Gamma_3\Gamma_4$. Schematically, the solutions are

$$\begin{aligned}
&\text{(i)} \quad (+,\pm,\pm,\pm,\pm)\,,\\
&\text{(ii)} \quad (-,-,+,\pm,\pm)\,,\\
&\text{(iii)} \quad (-,+,-,\pm,\pm)\,,\\
&\text{(iv)} \quad (-,-,-,\pm,\pm)\,,
\end{aligned} \tag{2.16}$$

where the first three entries denote the eigenvalues (± 1) of the operators $\Gamma_0\Gamma_9$, $i\Gamma_1\Gamma_2$ and $i\Gamma_3\Gamma_4$, respectively. The eigenvalues of the remaining two rotation operators $i\Gamma_5\Gamma_6$ and $i\Gamma_7\Gamma_8$ are, of course, arbitrary.[4] Our pp-wave will thus preserve at least 16, but at most 28, supersymmetries. There are a possible 12 supernumerary Killing spinors (cases (ii–iv)), annihilated by Γ_-, in addition to the usual 16 (case (i)), annihilated by Γ_+. Moreover, it would seem that this is the largest number of supersymmetries that will be preserved by such solutions, since switching on components of G_3 in additional directions will only lead to fewer solutions of (2.14).

Further conditions on the Killing spinors come from the variation of the gravitino. Using the analysis of ref. [18, 44] as a guide, we have

$$\begin{aligned}
D_a\varepsilon &= \Omega_a\varepsilon + \Lambda_a\varepsilon^*\,,\\
\Omega_a &= -\frac{i\mu}{4}\,(\Gamma_{1234} + \Gamma_{5678})\,\Gamma_{+}\Gamma_a\,,\\
\Lambda_a &= \frac{1}{32}\left(\xi_{ij}\Gamma_{a+ij} - 3G_{abc}\Gamma^{bc}\right).
\end{aligned} \tag{2.17}$$

Since $\Omega_v = \Lambda_v = 0$, the v component implies $\varepsilon = \varepsilon(u,x)$, and the i components give

$$\begin{aligned}
\partial_i\varepsilon &= \Omega_i\varepsilon + \Lambda_i\varepsilon^*\,,\\
\Omega_i &= -\frac{i\mu}{4}\,(\Gamma_{1234} + \Gamma_{5678})\,\Gamma_{+}\Gamma_i\,,\\
\Lambda_i &= \frac{1}{32}\,(\Gamma_i\,\not{\xi} - 8\xi_{ij}\Gamma_j)\,\Gamma_{+}\,.
\end{aligned} \tag{2.18}$$

Using the fact that $\Gamma_+^2 = 0$, we have

$$\Omega_i\Omega_j = \Lambda_i\Lambda_j = \Omega_i\Lambda_j = 0 \tag{2.19}$$

[4] Of course, the overall product of eigenvalues must be positive as implied by the chirality projection $\Gamma_{11}\varepsilon = \varepsilon$.

so that

$$\varepsilon = \chi(u) + x^i \rho_i(u)\,. \tag{2.20}$$

Differentiating with respect to x^i, one finds that $\rho_i = \Omega_i \chi + \Lambda_i \chi^*$, so

$$\varepsilon = \chi + x^i(\Omega_i \chi + \Lambda_i \chi^*)\,. \tag{2.21}$$

Note that the condition (2.14) now acts only on χ: $\not{\xi}\,\Gamma_+ \chi = 0$.

We are left with the u component of (2.17):

$$\begin{aligned}
\partial_u \varepsilon &= \frac{1}{2} A_{ij} x^i \Gamma_j \Gamma_+ \varepsilon + \Omega_u \varepsilon + \Lambda_u \varepsilon^*\,,\\
\Omega_u &= -\frac{i}{4}\mu\,(\Gamma_{1234} + \Gamma_{5678})\,\Gamma_+ \Gamma_-\,,\\
\Lambda_u &= -\frac{1}{32}\,\not{\xi}\,(\Gamma_+\Gamma_- + 2)\,.
\end{aligned} \tag{2.22}$$

Given the form (2.21) of the spinors, the constraint (2.22) may be divided into separate components by collecting the terms independent of x^i and those linear in each of the x^i. Making use of (2.19), the former may be written as:

$$\frac{\mathrm{d}\chi}{\mathrm{d}u} = -\frac{i}{2}\mu\,(\Gamma_{1234} + \Gamma_{5678})\,\chi - \frac{1}{32}\,\not{\xi}\,(\Gamma_+\Gamma_- + 2)\,\chi^*\,. \tag{2.23}$$

Using this result, the remaining linear terms can be reduced to eight algebraic constraints

$$\begin{aligned}
&\left(A_{ij}\Gamma_j\Gamma_+ + \mu^2 \Gamma_i \Gamma_+ - \frac{1}{4}\,\not{\xi}\,\Lambda_i^* + \frac{1}{8}\Lambda_i\,\not{\xi}^* \right)\chi +\\
&+ \left(-i\mu\{(\Gamma_{1234} + \Gamma_{5678})\,, \Lambda_i\} + \frac{1}{4}\,\not{\xi}\Omega_i + \frac{1}{8}\Omega_i\,\not{\xi} \right)\chi^* = 0\,,
\end{aligned} \tag{2.24}$$

where we have used the chirality condition $\Gamma_{11}\varepsilon = \varepsilon$ which implies, for example,

$$(\Gamma_{1234} + \Gamma_{5678})\,\Omega_i \varepsilon = -i\mu\Gamma_+\Gamma_i \varepsilon\,, \tag{2.25}$$

and related properties. After some elementary gamma matrix manipulations, we find that these constraints become

$$\left(A_{ij}\Gamma_j + \mu^2\Gamma_i - \frac{1}{32}\xi_{jk}\xi^*_{kl}\Gamma_i\Gamma_{jl} - \frac{1}{4}\xi^*_{ij}\xi_{jk}\Gamma_k \right)\Gamma_+\chi + \frac{i\mu}{8}\Gamma_{5678}\Gamma_i\,\not{\xi}\,\Gamma_+\chi^* = 0\,, \tag{2.26}$$

for each i, where we have made use of the fact that $\not{\xi}\Gamma_+\chi = 0$. Finally, we substitute explicitly for ξ_{ij}, from the Ansatz (2.10), giving

$$\begin{aligned}
&\Bigg(A_{ij}\Gamma_j + \mu^2\Gamma_i - \frac{1}{8}|\zeta|^2\Gamma_i\,(i\Gamma_{12} + i\Gamma_{34}) + \frac{1}{2}|\zeta|^2(\delta_{i1} + \delta_{i2})\Gamma_i(1 + i\Gamma_{12}) +\\
&+ \frac{1}{2}|\zeta|^2(\delta_{i3} + \delta_{i4})\Gamma_i(1 + i\Gamma_{34})\Bigg)\Gamma_+\chi + \frac{i}{4}\mu\zeta\,\Gamma_{5678}\Gamma_i\Gamma_{13}(1 + i\Gamma_{12})(1 + i\Gamma_{34})\,\Gamma_+\chi^* = 0\,.
\end{aligned} \tag{2.27}$$

From the above, it is clear that we recover the usual 16 Killing spinors, annihilated by Γ_+. The question remains which of the cases (ii–iv) in (2.16) also survive the projections (2.27) to become supernumerary supersymmetries. Note that cases (ii) and (iii) have equal and opposite eigenvalues of the rotation operators $i\Gamma_{12}$ and $i\Gamma_{34}$. The first has

$$(1 + i\Gamma_{12})\chi = 0 = (1 - i\Gamma_{34})\chi\,, \tag{2.28}$$

whereas the second has

$$(1 - i\Gamma_{12})\chi = 0 = (1 + i\Gamma_{34})\chi\,. \tag{2.29}$$

This leads to a simplification in (2.27). First, as these spinors satisfy

$$\not{\zeta}\,\Gamma_+\chi^* = 0\,. \tag{2.30}$$

the last term proportional to χ^* is eliminated. Further, the term proportional to $(i\Gamma_{12} + i\Gamma_{34})$ also vanishes. Thus (2.27) reduces to

$$\begin{aligned}\Bigg(A_{ij}\Gamma_j + \mu^2\Gamma_i + \frac{1}{2}|\zeta|^2(\delta_{i1} + \delta_{i2})\Gamma_i(1 + i\Gamma_{12}) +& \\ + \frac{1}{2}|\zeta|^2(\delta_{i3} + \delta_{i4})\Gamma_i(1 + i\Gamma_{34})\Bigg)\Gamma_+\chi = 0\,.&\end{aligned} \tag{2.31}$$

The spinors (ii) satisfying (2.28) must have

$$A_{ij} = -\mu^2\delta_{ij}\,, \qquad i,j \neq 1,2\,, \qquad A_{11} = A_{22} = -(\mu^2 + |\zeta|^2)\,, \tag{2.32}$$

whereas in case (iii) with (2.29), requires instead

$$A_{ij} = -\mu^2\delta_{ij}\,, \qquad i,j \neq 3,4\,, \qquad A_{33} = A_{44} = -\left(\mu^2 + |\zeta|^2\right)\,. \tag{2.33}$$

Thus, although we can have either one or the other, we cannot have both cases (ii) and (iii) as supernumerary supersymmetries. Taking case (ii) to be Killing, the metric is thus:

$$\mathrm{d}s^2 = 2\mathrm{d}u\mathrm{d}v - \left(\mu^2\sum_{i=3}^{8} x^i x^i + (\mu^2 + |\zeta|^2)|z^1|^2\right)\mathrm{d}u^2 + \mathrm{d}s^2\left(\mathbb{E}^8\right)\,. \tag{2.34}$$

Note that this does indeed solve the equation of motion (2.11), with F_5 and G_3 as in (2.10).

Finally we should consider case (iv) in (2.16). Explicitly substituting the corresponding eigenvalues for $i\Gamma_{12}$ and $i\Gamma_{34}$ into (2.27) yields

$$\left(A_{ij}\Gamma_j + \mu^2\Gamma_i - \frac{1}{4}|\zeta|^2\Gamma_i\right)\Gamma_+\chi + i\mu\zeta\,\Gamma_{5678}\Gamma_i\Gamma_{13}\,\Gamma_+\chi^* = 0\,. \tag{2.35}$$

While this equation has new non-trivial solutions for an appropriate choice of A_{ij}, we have already committed ourselves to that given in (2.32). Given this form of the metric, it is straightforward to show that the eight constraints (2.35) cannot be simultaneously satisfied. For example, consider multiplying the $i = 1$ component by Γ_{-1}

$$-\frac{5}{2}|\zeta|^2\chi + 2i\mu\zeta\,\Gamma_{135678}\chi^* = 0\,, \tag{2.36}$$

using $\Gamma_-\chi = 0$ for these particular spinors. Similarly the $i = 3$ component yields

$$-\frac{1}{2}|\zeta|^2\chi + 2i\mu\zeta\,\Gamma_{135678}\chi^* = 0\,. \tag{2.37}$$

Hence the difference of these two equations leaves $\chi = 0$. Therefore, all told, we have a solution which preserves 20 supersymmetries, which includes the standard 16 supersymmetries of case (i) and the 4 supernumerary Killing spinors of case (ii). All of the 20

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supersymmetries which this specific solution preserves depend on the null coordinate u since, from (2.24) we have

$$\frac{\mathrm{d}\chi}{\mathrm{d}u} = -\frac{i}{2}\mu\left(\Gamma_{1234} + \Gamma_{5678}\right)\chi - \frac{1}{8}\,\not{G}\,\chi^{*}\,, \tag{2.38}$$

which one may verify always has a non-trivial solution.

In the following section, we will see that precisely this pp-wave can be obtained as a Penrose limit of the supergravity solution exhibited by Pilch and Warner in ref. [35].

3. Taking the Penrose limit at the IR fixed point

The solution of five-dimensional $\mathcal{N}=8$ gauged supergravity presented in ref. [13] interpolates between two supersymmetric critical points of the scalar potential. In the ultra-violet (UV), it gives the standard maximally supersymmetric AdS_5 critical point, and in the IR it gives the $\mathcal{N}=2$ AdS_5 critical point found in ref. [45]. The solution provides a gravity dual of $\mathcal{N}=4$ SYM theory perturbed by a mass term for a single $\mathcal{N}=1$ chiral superfield [13, 46], which we take to be Φ_3. It thus describes the $\mathcal{N}=1$ RG flow between the $\mathcal{N}=4$ theory and the IR fixed point, which is a large-N limit of the superconformal $\mathcal{N}=1$ theory of Leigh and Strassler [47]. On the gravity side, one has a solution which preserves 1/8 of the supersymmetries everywhere, this being enhanced to 1/4 at the IR fixed point.

Since the five-dimensional theory is believed to be a consistent truncation of type-IIB supergravity [45, 48], one can lift this solution directly to ten dimensions. The resulting $\mathcal{N}=1$ geometry [35, 39] interpolates between $AdS_5\times S^5$ in the UV, and a warped product of another AdS_5 with a squashed five-sphere in the IR. Before turning to the flow geometry, let us first restrict ourselves to the IR fixed point solution of ref. [35], which is the gravity dual of the $\mathcal{N}=1$ superconformal theory in its own right, i.e. it does not represent the process of deforming away from the UV $\mathcal{N}=4$ fixed point gauge theory.

For completeness, and to fix our conventions, we present this fixed point solution in appendix A. As discussed therein, we want to work in coordinates for which the $\mathrm{U}(1)_R$ symmetry is simplest so we shift the S^3 Euler angle $\beta\to\beta+2\phi$, to give a solution with a global $\mathrm{U}(1)_R = \mathrm{U}(1)_\phi$ symmetry. Performing this coordinate transformation on the solution (A.14), (A.15) and (A.17) and writing the AdS space in global coordinates gives

$$\mathrm{d}s^2 = L^2\Omega^2\left(-\cosh^2\rho\,\mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \sinh^2\rho\,\mathrm{d}\Omega_3^2\right) + \mathrm{d}s_5^2\,, \tag{3.1}$$

$$\begin{aligned}\mathrm{d}s_5^2 = \frac{2}{3}L^2\Omega^2\Bigg[&\mathrm{d}\theta^2 + \frac{4\cos^2\theta}{(3-\cos 2\theta)}(\sigma_1^2+\sigma_2^2) + \frac{4\sin^2(2\theta)}{(3-\cos 2\theta)^2}\,(\sigma_3+\mathrm{d}\phi)^2 + \\ &+\frac{8}{3}\left(\frac{2\sin^2\theta-\cos^2\theta}{3-\cos(2\theta)}\right)^2\left(\mathrm{d}\phi - \frac{2\cos^2\theta}{2\sin^2\theta-\cos^2\theta}\sigma_3\right)^2\Bigg],\end{aligned} \tag{3.2}$$

$$F_5 = -\frac{2^{5/3}}{3}L^4\cosh\rho\,\ \sinh^3\rho\,(1+\star)\,\mathrm{d}\tau\wedge\mathrm{d}\rho\wedge\epsilon(S^3)\,, \tag{3.3}$$

$$\begin{aligned}G_3 = -iL_0^2\cos\theta\Bigg[&\mathrm{d}\theta\wedge\mathrm{d}\phi - \frac{8\cos^2\theta}{(3-\cos(2\theta))^2}\mathrm{d}\theta\wedge(\sigma_3+\mathrm{d}\phi) - \\ &-\frac{2i\sin(2\theta)}{(3-\cos(2\theta))}\sigma_3\wedge\mathrm{d}\phi\Bigg]\wedge(\sigma_1+i\sigma_2)\,.\end{aligned} \tag{3.4}$$

where

$$\Omega^2 = \frac{2^{1/3}}{\sqrt{3}} \sqrt{3 - \cos(2\theta)}\,, \tag{3.5}$$

and the AdS radius, L, is given in terms L_0, the AdS radius of the UV spacetime, by

$$L = \frac{3}{2^{5/3}} L_0\,. \tag{3.6}$$

Note that the three-form field strength G_3 could include an arbitrary constant phase, which we have set to -1 here. The global isometry group of the metric is $\mathrm{SU}(2) \times \mathrm{U}(1)_\beta \times \mathrm{U}(1)_\phi$, where $\mathrm{U}(1)_\beta$ denotes the shift in the Euler angle β, rotating σ_1 into σ_2. However, from the three-form one sees that the $\mathrm{U}(1)_R$ R-symmetry of the solution as a whole is $\mathrm{U}(1)_R = \mathrm{U}(1)_\phi$, as required.

Since we are working in global coordinates, we can consider the simple null geodesics for which $\rho = 0$. An examination of the θ geodesic equation shows that one can also consistently set either $\theta = 0$ or $\theta = \pi/2$. As discussed in appendix A, the moduli space of a D3-brane probe in this geometry [49, 50] corresponds to $\theta = 0$, whereas the massive direction away from the moduli space corresponds to $\theta = \pi/2$. We will consider taking the Penrose limit along a null geodesic in the moduli space, with $\theta = 0$. We do also consider the other class of geodesics, with $\theta = \pi/2$, but since we have not been able to see the relevance of the latter to the gauge theory, we have consigned the analysis of this case to appendix B. Suffice it to say here that we do not understand the significance in the gauge theory of considering geodesics with angular momentum in the massive direction, since it is precisely this direction which is to be "integrated out" in the $\mathcal{N} = 1$ gauge theory at the IR fixed point; motion in this direction (and hence gauge theory operator excitations) should be effectively frozen in the low energy field theory.

We thus take $\theta = 0$, in which case the effective lagrangian is

$$\mathcal{L} = L^2 \Omega_0^2 \left[-\dot\tau^2 + \frac{1}{3}\left(\dot\alpha^2 + \sin^2\alpha\,\dot\gamma^2\right) + \frac{4}{9}\left(\dot\phi + \dot\beta + \cos\alpha\,\dot\gamma\right)^2 \right], \tag{3.7}$$

where $\Omega_0^2 = 2^{1/3}\sqrt{2/3}$ and a dot denotes differentiation with respect to the affine parameter. We can thus also consider geodesics for which $\alpha = 0$, giving[5]

$$\mathcal{L} = L^2 \Omega_0^2 \left(-\dot\tau^2 + \frac{4}{9}\dot\psi^2 \right), \tag{3.8}$$

where we have defined

$$\psi = \phi + \beta + \gamma\,, \tag{3.9}$$

to be the direction in which our geodesics have an angular momentum. The natural light-cone coordinates are then

$$u = \frac{1}{2E}\left(\tau + \frac{2}{3}\psi\right), \qquad v = -EL^2\Omega_0^2\left(\tau - \frac{2}{3}\psi\right), \tag{3.10}$$

[5] Although more general geodesics could be considered here, we suspect that other geodesics would simply give a hamiltonian with an alternate linear combination of the charges J and J_3 (to be defined below).

where E is the conserved energy associated with the Killing vector $\partial/\partial\tau$. If h is the conserved angular momentum associated with the Killing vector $\partial/\partial\psi$, we have $E = (2/3)h$. We implement the fact that we are considering $\rho = \theta = \alpha = 0$ geodesics by taking

$$\rho = \frac{r}{L}\,, \qquad \theta = \frac{y}{L}\,, \qquad \alpha = \frac{w}{L}\,, \tag{3.11}$$

and considering the $L \to \infty$ limit.

Dropping terms of $\mathcal{O}(1/L^2)$, defining two new angular coordinates as

$$\hat{\phi} = \phi - \frac{1}{3}\psi\,, \qquad \hat{\gamma} = \gamma - \frac{2}{3}\psi\,, \tag{3.12}$$

and rescaling r, y and w, the metric becomes

$$\mathrm{d}s^2 = 2\mathrm{d}u\mathrm{d}v - E^2\left(r^2 + w^2 + 4y^2\right)\mathrm{d}u^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega_3^2 + \mathrm{d}y^2 + y^2\mathrm{d}\hat{\phi}^2 + \mathrm{d}w^2 + w^2\mathrm{d}\hat{\gamma}^2\,. \tag{3.13}$$

With the same definitions of $\hat{\phi}$ and $\hat{\gamma}$, and rescalings of coordinates, taking the Penrose limit of the form fields gives

$$\begin{aligned} F_5 &= -E\,(1+\star)\,\mathrm{d}u \wedge \epsilon\left(\mathbb{E}^4\right)\,, \\ G_3 &= -\sqrt{3}Ee^{i\beta}\mathrm{d}u \wedge \left(\mathrm{d}y - iy^2\mathrm{d}\hat{\phi}\right) \wedge \left(\mathrm{d}w - iw^2\mathrm{d}\hat{\gamma}\right)\,, \\ &= \sqrt{3}E\;\mathrm{d}u \wedge \mathrm{d}z^1 \wedge \mathrm{d}z^2\,, \end{aligned} \tag{3.14}$$

where $\epsilon(\mathbb{E}^4) = r^3\mathrm{d}r \wedge \epsilon(S^3)$ and we have defined complex coordinates on the remaining $\mathbb{E}^4$. It should be clear that the resulting solution is precisely the 5/8 supersymmetric pp-wave considered in the previous section, with $\mu = -E$ and $\zeta = \sqrt{3}E$. The Penrose limit we have considered thus enhances supersymmetry from 1/4 to 5/8.

This is a genuinely new background, and we have a specific identification of it as dual to a known four-dimensional gauge theory. Given that this is so, we now turn to an analysis of string theory in this background. It should be clear that not all of the worldsheet scalars will have equal masses, as is the standard case [19]: the factor of 4 in the matrix A_{ij} of the metric (3.13) implies that, whereas six of the scalars will have a mass proportional to E, the remaining two will have a mass proportional to $2E$. This will have non-trivial consequences for the operator spectrum of the gauge theory as well, as we shall see.

4. String propagation

4.1 World-sheet analysis: bosonic sector

The fields which will contribute to our discussion of the world-sheet bosons are the NS-NS fields, i.e. the metric and the antisymmetric tensor field B_2. There are a number of useful choices for a gauge within which to work with the B-field, with (of course) the same resulting physics. A convenient choice for our purposes is

$$B_2 = -\sqrt{3}E\left(x^1\mathrm{d}u \wedge \mathrm{d}x^3 - x^2\mathrm{d}u \wedge \mathrm{d}x^4\right)\,. \tag{4.1}$$

The relevant part of the world-sheet action is:

$$S_B = -\frac{1}{4\pi\alpha'}\int \mathrm{d}\sigma\mathrm{d}\tau \left\{\sqrt{-g}g^{\alpha\beta}\left(2\partial_\alpha U\partial_\beta V + A_{ij}X^iX^j\partial_\alpha U\partial_\beta U + \partial_\alpha X^i\partial_\beta X^i\right) - \right.$$
$$\left. - 2\sqrt{3}E\epsilon^{\alpha\beta}\left(X^1\partial_\alpha U\partial_\beta X^3 - X^2\partial_\alpha U\partial_\beta X^4\right)\right\}, \tag{4.2}$$

where $\epsilon^{01} = 1$ and we shall use the familiar world-sheet gauge choice $g_{\alpha\beta} = \eta_{\alpha\beta}$. We have used world-sheet coordinates σ^α, where $\alpha, \beta = 0, 1$, and $\sigma^0 = \tau, \sigma^1 = \sigma$. A_{ij} may be read off from (3.13).

Variation of V gives rise to the equation of motion for U, namely $\nabla^2 U = 0$. So we can work in the standard light-cone gauge with $U = \alpha' p^+\tau + \text{constant}$. In that case, the worldsheet scalars obey the following equations:

$$\begin{aligned}
\nabla^2X^1 - 4M^2X^1 + \sqrt{3}M\partial_\sigma X^3 &= 0\,,\\
\nabla^2X^2 - 4M^2X^2 - \sqrt{3}M\partial_\sigma X^4 &= 0\,,\\
\nabla^2X^3 - M^2X^3 - \sqrt{3}M\partial_\sigma X^1 &= 0\,,\\
\nabla^2X^4 - M^2X^4 + \sqrt{3}M\partial_\sigma X^2 &= 0\,,\\
\nabla^2X^p - M^2X^p &= 0\,,
\end{aligned} \tag{4.3}$$

where $p, q = 5, 6, 7, 8$ will label the directions which are unaffected by the B-field, and where we have set $M = E\alpha' p^+$. The structure of these equations is interesting: they are roughly familiar from other pp-wave systems (see for example refs. [36, 37, 38], which follow on from refs. [51]–[54]), but there are crucial differences brought on by the asymmetry between the 1–3 plane and the 2–4 plane (visible in (3.13)), which will produce an amusing mass splitting in the spectrum, as we shall see. The two independent components of the standard constraint from world-sheet reparameterisations, $T_{\alpha\beta} = 0$, are

$$\partial_\sigma V = -\frac{1}{\alpha' p^+}\partial_\tau X^i\partial_\sigma X^i, \tag{4.4}$$

$$\partial_\tau V = -\frac{1}{2\alpha' p^+}\left(\partial_\tau X^i\partial_\tau X^i + \partial_\sigma X^i\partial_\sigma X^i + (\alpha' p^+)^2A_{ij}X^iX^j\right), \tag{4.5}$$

which allow for the elimination of V in the usual way. Integrating the former over σ gives

$$\int_0^{2\pi}\mathrm{d}\sigma\,\partial_\tau X^i\partial_\sigma X^i = 0\,. \tag{4.6}$$

In the light-cone gauge, the action becomes

$$S_B = -\frac{1}{4\pi\alpha'}\int \mathrm{d}\sigma\mathrm{d}\tau\left\{-2\alpha' p^+\partial_\tau V - (\alpha' p^+)^2A_{ij}X^iX^j + \eta^{\alpha\beta}\partial_\alpha X^i\partial_\beta X^i - \right.$$
$$\left. - \; 2\sqrt{3}M\left(X^1\partial_\sigma X^3 - X^2\partial_\sigma X^4\right)\right\}, \tag{4.7}$$

from which it is easy to derive the hamiltonian:

$$H_B = \frac{1}{4\pi\alpha'}\int_0^{2\pi}\mathrm{d}\sigma\left\{\left(2\pi\alpha'\right)^2\Pi^i\Pi^i - \left(\alpha' p^+\right)^2A_{ij}X^iX^j + \partial_\sigma X^i\partial_\sigma X^i - \right.$$
$$\left. - 2\sqrt{3}M\left(X^1\partial_\sigma X^3 - X^2\partial_\sigma X^4\right)\right\}, \tag{4.8}$$

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where the conjugate variable to X^i is

$$\Pi^i = \frac{1}{2\pi\alpha'}\partial_\tau X^i \,. \tag{4.9}$$

To solve for the eigenmodes of the system, subject to the usual periodic boundary conditions $X^i(\tau, \sigma + 2\pi) = X^i(\tau, \sigma)$, we Fourier expand

$$X^i(\tau, \sigma) = \sum_n C_n^i e^{i(\omega_n \tau + n\sigma)} \,, \tag{4.10}$$

from which we get the following system of equations:

$$\begin{aligned}
\left[-\omega_n^2 + (n^2 + 4M^2)\right] C_n^1 - in\sqrt{3}MC_n^3 &= 0\,, \\
\left[-\omega_n^2 + (n^2 + 4M^2)\right] C_n^2 + in\sqrt{3}MC_n^4 &= 0\,, \\
\left[-\omega_n^2 + (n^2 + M^2)\right] C_n^3 + in\sqrt{3}MC_n^1 &= 0\,, \\
\left[-\omega_n^2 + (n^2 + M^2)\right] C_n^4 - in\sqrt{3}MC_n^2 &= 0\,, \\
\left[-\omega_n^2 + (n^2 + M^2)\right] C_n^p &= 0\,,
\end{aligned} \tag{4.11}$$

for some unknown coefficients C_n^i. For the X^p, the normal modes are

$$\omega_n^2 = n^2 + M^2 \,. \tag{4.12}$$

As is by now well-known, a key feature of this spectrum is that even the zero modes ($n = 0$) have an oscillator frequency $\omega_0 = M$ set by the pp-wave background, corresponding to the mass of the world-sheet bosons associated with those directions. Beyond that, there are simply four independent towers of oscillators (one for each direction) with the mode expansion for these coordinates being [36]

$$X^p(\tau, \sigma) = \cos M\tau x_0^p + \frac{\alpha'}{M}\sin M\tau p_0^p + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{\omega_n}\left(\alpha_n^p e^{in\sigma} + \tilde{\alpha}_n^p e^{-in\sigma}\right)e^{-i\omega_n\tau}, \tag{4.13}$$

where, to ensure reality, we have

$$\omega_n = \sqrt{n^2 + M^2}\,, \qquad (n > 0)\,, \qquad \omega_n = -\sqrt{n^2 + M^2}\,, \qquad (n < 0)\,, \tag{4.14}$$

and

$$(\alpha_n^p)^\dagger = \alpha_{-n}^p\,, \qquad (\tilde{\alpha}_n^p)^\dagger = \tilde{\alpha}_{-n}^p\,. \tag{4.15}$$

The non-vanishing Poisson brackets are easily found to be

$$[x_0^p, p_0^q]_{PB} = \delta^{pq}\,, \qquad [\alpha_m^p, \alpha_n^q]_{PB} = [\tilde{\alpha}_m^p, \tilde{\alpha}_n^q]_{PB} = -i\omega_m\delta_{m+n,0}\delta^{pq}\,. \tag{4.16}$$

Turning to the directions $i = 1, 2, 3, 4$, we have a more complicated system, since the masses in the 1 and 3, and 2 and 4, directions are different. We do not expect a simple symmetric result for the coupled system. After a bit of algebra, we obtain the intriguing formula:

$$\omega_n^2 = \frac{1}{2}\left(2n^2 + 5M^2 \pm \sqrt{12n^2M^2 + 9M^4}\right) \equiv \left(\omega_n^\pm\right)^2\,, \tag{4.17}$$

for these directions. This will give *two* distinct pairs of frequencies, giving again four independent families of oscillators. We observe that the natural frequencies of the zero modes are $\omega_0^- = \omega_0 = M$ and $\omega_0^+ = 2M$, as expected. Explicitly, the mode expansions for these coordinates are thus

$$\begin{aligned} X^1(\tau,\sigma) &= \cos 2M\tau x_0^1 + \frac{\alpha'}{2M}\sin 2M\tau p_0^1 + \\ &+ i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\left[\frac{1}{\omega_n^+}\left(\beta_n^1 e^{in\sigma} + \tilde{\beta}_n^1 e^{-in\sigma}\right)e^{-i\omega_n^+\tau} + \frac{1}{\omega_n^-}\left(\gamma_n^1 e^{in\sigma} + \tilde{\gamma}_n^1 e^{-in\sigma}\right)e^{-i\omega_n^-\tau}\right], \end{aligned} \tag{4.18}$$

$$\begin{aligned} X^3(\tau,\sigma) &= \cos M\tau x_0^3 + \frac{\alpha'}{M}\sin M\tau p_0^3 + \\ &+ i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\left[\frac{c_n^+}{\omega_n^+}\left(\beta_n^1 e^{in\sigma} - \tilde{\beta}_n^1 e^{-in\sigma}\right)e^{-i\omega_n^+\tau} + \frac{c_n^-}{\omega_n^-}\left(\gamma_n^1 e^{in\sigma} - \tilde{\gamma}_n^1 e^{-in\sigma}\right)e^{-i\omega_n^-\tau}\right], \end{aligned} \tag{4.19}$$

where $\omega_n^\pm$ is given by the positive (negative) root of (4.17) for positive (negative) n and

$$c_n^\pm = \frac{i}{2\sqrt{3}nM}\left(-3M^2 \pm \sqrt{12n^2M^2 + 9M^4}\right), \tag{4.20}$$

which obeys $c_n^+ c_n^- = 1$. Similar expressions hold for X^2 and X^4, with $\{\beta^1,\gamma^1,\tilde{\beta}^1,\tilde{\gamma}^1\}$ replaced by $\{\beta^2,\gamma^2,\tilde{\beta}^2,\tilde{\gamma}^2\}$ and $c_n^\pm$ replaced by $-c_n^\pm$. With $A = 1,2$, reality of the coordinates implies

$$\left(\beta_n^A\right)^\dagger = \beta_{-n}^A\,, \qquad \left(\gamma_n^A\right)^\dagger = \gamma_{-n}^A\,, \qquad \left(\tilde{\beta}_n^A\right)^\dagger = \tilde{\beta}_{-n}^A\,, \qquad \left(\tilde{\gamma}_n^A\right)^\dagger = \tilde{\gamma}_{-n}^A\,. \tag{4.21}$$

In addition to (4.16), the remaining non-vanishing Poisson brackets are then

$$\begin{aligned} \left[x_0^A, p_0^B\right]_{PB} &= \left[x_0^{A+2}, p_0^{B+2}\right]_{PB} = \delta^{AB}\,, \\ \left[\beta_m^A, \beta_n^B\right]_{PB} &= \left[\tilde{\beta}_m^A, \tilde{\beta}_n^B\right]_{PB} = -i\omega_m^+ \frac{c_m^-}{c_m^- - c_m^+}\delta_{m+n,0}\delta^{AB}\,, \\ \left[\gamma_m^A, \gamma_n^B\right]_{PB} &= \left[\tilde{\gamma}_m^A, \tilde{\gamma}_n^B\right]_{PB} = -i\omega_m^- \frac{c_m^+}{c_m^+ - c_m^-}\delta_{m+n,0}\delta^{AB}\,. \end{aligned} \tag{4.22}$$

The constraint (4.6) becomes $N = \tilde{N}$, where

$$\begin{aligned} N &= \sum_{n\neq 0} n\left[\frac{1}{\omega_n}\alpha_{-n}^p\alpha_n^p + \frac{1}{\omega_n^+}\left(1 - c_n^{+2}\right)\beta_{-n}^A\beta_n^A + \frac{1}{\omega_n^-}\left(1 - c_n^{-2}\right)\gamma_{-n}^A\gamma_n^A\right], \\ \tilde{N} &= \sum_{n\neq 0} n\left[\frac{1}{\omega_n}\tilde{\alpha}_{-n}^p\tilde{\alpha}_n^p + \frac{1}{\omega_n^+}\left(1 - c_n^{+2}\right)\tilde{\beta}_{-n}^A\tilde{\beta}_n^A + \frac{1}{\omega_n^-}\left(1 - c_n^{-2}\right)\tilde{\gamma}_{-n}^A\tilde{\gamma}_n^A\right], \end{aligned} \tag{4.23}$$

and the hamiltonian (4.8) is

$$\begin{aligned} H_B = \frac{1}{2\alpha'}&\left(\alpha'^2 p_0^i p_0^i + 4M^2\sum_{i=1,2} x_0^i x_0^i + M^2 \sum_{i=3}^{8} x_0^i x_0^i\right) + \\ &+ \frac{1}{2}\sum_{n\neq 0}\left(\alpha_{-n}^p\alpha_n^p + \tilde{\alpha}_{-n}^p\tilde{\alpha}_n^p + \left(1 - c_n^{+2}\right)\left(\beta_{-n}^A\beta_n^A + \tilde{\beta}_{-n}^A\tilde{\beta}_n^A\right) + \left(1 - c_n^{-2}\right)\left(\gamma_{-n}^A\gamma_n^A + \tilde{\gamma}_{-n}^A\tilde{\gamma}_n^A\right)\right). \end{aligned} \tag{4.24}$$

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To quantise the system, we replace the Poisson brackets with commutators in the usual way. We further take, for $n > 0$,

$$
\begin{aligned}
\alpha_n^p &= \sqrt{\omega_n} a_n^p , & \alpha_{-n}^p &= \sqrt{\omega_n} \bar{a}_n^p , \\
\beta_n^A &= \sqrt{\omega_n^+ \frac{c_n^-}{c_n^- - c_n^+}} b_n^A , & \beta_{-n}^A &= \sqrt{\omega_n^+ \frac{c_n^-}{c_n^- - c_n^+}} \bar{b}_n^A , \\
\gamma_n^A &= \sqrt{\omega_n^- \frac{c_n^+}{c_n^+ - c_n^-}} c_n^A , & \gamma_{-n}^A &= \sqrt{\omega_n^- \frac{c_n^+}{c_n^+ - c_n^-}} \bar{c}_n^A ,
\end{aligned}
\tag{4.25}
$$

and similarly for the the independent set of operators with a tilde, and combine the zero modes as

$$
a_0^i = \frac{1}{\sqrt{4M\alpha'}} \left(\alpha' p_0^i - 2iMx_0^i \right) , \quad \bar{a}_0^i = \frac{1}{\sqrt{4M\alpha'}} \left(\alpha' p_0^i + 2iMx_0^i \right) , \qquad (i = 1, 2) , \tag{4.26}
$$

$$
a_0^i = \frac{1}{\sqrt{2M\alpha'}} \left(\alpha' p_0^i - iMx_0^i \right) , \quad \bar{a}_0^i = \frac{1}{\sqrt{2M\alpha'}} \left(\alpha' p_0^i + iMx_0^i \right) , \qquad (i = 3, \ldots, 8) . \tag{4.27}
$$

The new creation and annihilation operators obey the standard harmonic oscillator commutation relations

$$
\left[a_0^i, \bar{a}_0^j \right] = \delta^{ij} , \qquad [a_m^p, \bar{a}_n^q] = \delta_{mn} \delta^{pq} , \qquad [b_n^A, \bar{b}_m^B] = [c_n^A, \bar{c}_m^B] = \delta_{nm} \delta^{AB} , \tag{4.28}
$$

and similarly for the tilded set of operators. In this basis, the hamiltonian (4.25) becomes

$$
H = \Delta E + 2M \sum_{i=1,2} N_0^{(i)} + M \sum_{i=3}^{8} N_0^{(i)} + \sum_{n>0} \left(\omega_n N_n^{(a)} + \omega_n^+ N_n^{(b)} + \omega_n^- N_n^{(c)} \right) , \tag{4.29}
$$

where ω_n and $\omega_n^\pm$ are given by (4.12) and (4.17), respectively, and ΔE is the zero point energy. The occupation numbers are given by

$$
N_n^{(a)} = \bar{a}_n^p a_n^p + \tilde{\bar{a}}_n^p \tilde{a}_n^p , \tag{4.30}
$$

and similarly for $N_n^{(b)}$ and $N_n^{(c)}$, and we have defined

$$
N_0^{(i)} = \bar{a}_0^i a_0^i . \tag{4.31}
$$

The spectrum of the bosonic string is thus of the same form as in the maximally supersymmetric case [19, 21, 36], the only difference being the more complicated frequencies $\omega_n^\pm$. We will see shortly that precisely the same frequencies appear in the normal modes of some of the fermions.

At this point, however, the interesting structure of our equations leads us to consider a slightly more general class of scenarios in appendix C. There, we comment briefly on an interesting property of the equations of motion for the directions affected by the B-field, pointing to the possibility of a new type of stringy instability.

4.2 World-sheet analysis: fermionic sector

The problem as to how to include R-R fields in the worldsheet analysis of the superstring is a difficult one. Techniques utilising coset superspaces have been used in an attempt to construct actions for superstrings in AdS backgrounds (e.g. ref. [55]), although the resulting action is difficult to quantise explicitly. More recently, such techniques have been applied [19, 21] to the maximally supersymmetric pp-wave background of ref. [18]. In the light-cone gauge, the superstring action in this background simplifies considerably and can, in fact, be quantised: it turns out the five-form field strength only gives rise to mass terms for the fermions.[6] More heuristically, since this background admits a null Killing vector, it can be argued [36] that the fermionic action is a direct covariantisation of the flat action, at least in the standard light-cone gauge.

In our conventions, the light-cone gauge is implemented *via*

$$\Gamma^{-}\theta = \Gamma_{+}\theta = 0\,, \tag{4.32}$$

in which case the fermionic action is simply [36]

$$S_F = \frac{i}{\pi}\int \mathrm{d}\sigma \mathrm{d}\tau \left(\eta^{\alpha\beta}\delta_{IJ} - \epsilon^{\alpha\beta}\rho_{IJ}\right)\partial_\alpha X^a \partial_\beta X^b \bar{\theta}^I \Gamma_a \mathcal{D}_b \theta^J\,, \tag{4.33}$$

where $I, J = 1, 2$ denote the two 16-component Majorana-Weyl spinors. It is important to realise that the supercovariant derivative appearing in this action is written in the string, as opposed to the Einstein, frame. Since the dilaton is zero, we have not had to worry about this distinction so far. However, our analysis of section 2.2 was performed in the Einstein frame and, since the gravitino is shifted by a multiple of the dilatino when going from one frame to the other [65], what we mean by the supercovariant derivative is also changed. In terms of the Pauli matrices, τ_i, the two-dimensional gamma matrices are $\rho^0 = i\tau_2$ and $\rho^1 = \tau_1$, so that $\rho = \rho^0\rho^1 = \tau_3$. With $G_3 = H_3 + iF_3$, and viewed as acting on a column matrix, the supercovariant derivative then takes the form [36]

$$\mathcal{D}_a = D_a + \frac{1}{8}H_a{}^{bc}\Gamma_{bc}\rho + \frac{1}{48}F_{bcd}\Gamma^{bcd}\Gamma_a\rho_1 + \frac{1}{480}F_{b_1\ldots b_5}\Gamma^{b_1\ldots b_5}\Gamma_a\rho_0\,, \tag{4.34}$$

where

$$\not{H}_a = H_{bcd}\Gamma_a{}^{bcd} - 9H_{abc}\Gamma^{bc}\,, \tag{4.35}$$

and likewise for $\not{F}_a$. In the light-cone gauge, the action simplifies considerably, and we have [31, 36]

$$\begin{aligned} S_F = -\frac{i}{\pi}\alpha' p^+ \int \mathrm{d}\sigma \mathrm{d}\tau \Big\{ & \bar{\theta}\Gamma_-\left(\partial_\tau\theta + \rho\,\partial_\sigma\theta\right) + \frac{1}{8}\alpha' p^+ \bar{\theta}\Gamma_-\,\not{H}\rho\theta + \\ & + \frac{1}{8}\alpha' p^+ \bar{\theta}\Gamma_-\,\not{F}_3\rho_1\theta + \frac{1}{240}\alpha' p^+ \bar{\theta}\Gamma_-\,\not{F}_5\rho_0\theta \Big\}\,, \end{aligned} \tag{4.36}$$

[6] Other techniques can be used to derive the relevant action [31]: since the eleven-dimensional supermembrane action is known to $\mathcal{O}(\theta^2)$ [56], dimensional reduction will give rise [57] to the superstring action to the same order in the fermions; and this is all that is required in the case at hand.

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where now

$$\not{H} = H_{uij}\Gamma_{ij}\,, \qquad \not{F}_3 = F_{uij}\Gamma_{ij}\,, \qquad \not{F}_5 = F_{uijkl}\Gamma_{ijkl}\,. \tag{4.37}$$

We should note that the NS-NS three-form gives rise to a chiral interaction, whereas the R-R three-form field strength gives further mass terms [36].

Finally, then, we substitute for $H_{u13} = -H_{u24} = F_{u14} = F_{u23} = \sqrt{3}E$ and $F_{u1234} = F_{u5678} = -E$, and rewrite in terms of θ^1 and θ^2, giving

$$\begin{aligned} S_F = -\frac{i}{\pi}\alpha' p^+ \int \mathrm{d}\sigma\mathrm{d}\tau \Bigg\{ & \theta^1\Gamma_-\partial_+\theta^1 + \theta^2\Gamma_-\partial_-\theta^2 + \\ & + \frac{\sqrt{3}}{2}M\theta^1\Gamma_-(\Gamma_{14}+\Gamma_{23})\theta^2 + \frac{\sqrt{3}}{4}M\theta^1\Gamma_-(\Gamma_{13}-\Gamma_{24})\theta^1 - \\ & - \frac{\sqrt{3}}{4}M\theta^2\Gamma_-(\Gamma_{13}-\Gamma_{24})\theta^2 - 2M\theta^1\Gamma_-\Gamma_{1234}\theta^2 \Bigg\}\,, \end{aligned} \tag{4.38}$$

where

$$\partial_\pm = \partial_\tau \pm \partial_\sigma\,. \tag{4.39}$$

The equations of motion for θ^1 and θ^2 are then

$$\begin{aligned} \partial_+\theta^1 - M\Gamma_{1234}\theta^2 + \frac{\sqrt{3}}{4}M(\Gamma_{14}+\Gamma_{23})\theta^2 + \frac{\sqrt{3}}{4}M(\Gamma_{13}-\Gamma_{24})\theta^1 &= 0\,, \\ \partial_-\theta^2 + M\Gamma_{1234}\theta^1 + \frac{\sqrt{3}}{4}M(\Gamma_{14}+\Gamma_{23})\theta^1 - \frac{\sqrt{3}}{4}M(\Gamma_{13}-\Gamma_{24})\theta^2 &= 0\,. \end{aligned} \tag{4.40}$$

The next step is again to Fourier expand

$$\theta^I(\tau,\sigma) = \sum_n \theta_n^I(\tau)e^{in\sigma},$$

giving

$$\begin{aligned} \dot\theta_n^1 + M\left(\frac{\sqrt{3}}{4}(\Gamma_{14}+\Gamma_{23}) - \Gamma_{1234}\right)\theta_n^2 + \left(\frac{\sqrt{3}}{4}M(\Gamma_{13}-\Gamma_{24}) + in\right)\theta_n^1 &= 0\,, \\ \dot\theta_n^2 + M\left(\frac{\sqrt{3}}{4}(\Gamma_{14}+\Gamma_{23}) + \Gamma_{1234}\right)\theta_n^1 - \left(\frac{\sqrt{3}}{4}M(\Gamma_{13}-\Gamma_{24}) + in\right)\theta_n^2 &= 0\,. \end{aligned} \tag{4.41}$$

Differentiating with respect to τ and using (4.41) again to eliminate the first derivatives, results in

$$\ddot\varepsilon_n + A_n\varepsilon_n = 0\,, \tag{4.42}$$

where

$$A_n = \left(n^2 + \frac{7M^2}{4}\right)I - \frac{3M^2}{4}\Gamma_{1234} - \frac{i\sqrt{3}Mn}{2}(\Gamma_{13}-\Gamma_{24}) - \frac{i3M^2}{4}(\Gamma_{12}+\Gamma_{34})\,, \tag{4.43}$$

and we have re-combined θ^1 and θ^2 into a single complex spinor $\varepsilon = \theta^1 + i\theta^2$. In order to solve (4.42) we need to find the eigenspinors of the matrix A_n. To do this, we consider the constant spinors in the Chevalier basis as in section 2.2. In particular, we focus on the eigenvalues of $i\Gamma_{12}$ and $i\Gamma_{34}$, and denote the spinors as $\varepsilon^{\pm\pm}$, where

$$\begin{aligned} i\Gamma_{12}\varepsilon^{\pm(\cdot)} &= \pm\varepsilon^{\pm(\cdot)}\,, \\ i\Gamma_{34}\varepsilon^{(\cdot)\pm} &= \pm\varepsilon^{(\cdot)\pm}\,. \end{aligned} \tag{4.44}$$

so that

$$\begin{aligned} A_n\varepsilon^{++} &= (M^2+n^2)\varepsilon^{++} - \sqrt{3}Mn\varepsilon^{--}\,, \\ A_n\varepsilon^{+-} &= (M^2+n^2)\varepsilon^{+-}\,, \\ A_n\varepsilon^{-+} &= (M^2+n^2)\varepsilon^{-+}\,, \\ A_n\varepsilon^{--} &= (4M^2+n^2)\varepsilon^{--} - \sqrt{3}Mn\varepsilon^{++}\,. \end{aligned} \tag{4.45}$$

Therefore, at each level n, there are four fermionic oscillators with frequency given by

$$\omega_n^2 = n^2 + M^2\,, \tag{4.46}$$

and there are four fermionic oscillators with frequencies

$$\left(\omega_n^{\pm}\right)^2 = \frac{1}{2}\left(2n^2 + 5M^2 \pm \sqrt{12n^2M^2 + 9M^4}\right). \tag{4.47}$$

These exactly match the frequencies (4.12) and (4.17) found for the bosonic oscillators above, as presumably required by supersymmetry.

The mode expansions of the fermions are not particularly enlightening, and we will not exhibit them here. Suffice it to say that, given the above results concerning the frequencies of the fermion modes, we fully expect the total hamiltonian to be of the same form as the purely bosonic result (4.29). That is,

$$H = \Delta E + 2M\sum_{i=1,2} N_0^{(i)} + M\sum_{i=3}^{8} N_0^{(i)} + \sum_{n>0}\left(\omega_n N_n^1 + \omega_n^+ N_n^2 + \omega_n^- N_n^3\right), \tag{4.48}$$

where, in analogy with ref. [36], the zero point energy $\Delta E = (6\times 1/2 + 2\times 2\times 1/2)M = 5M$, the fermion zero modes appear in $N_0^{(i)}$ and the level operators $N_n^{1,2,3}$ now also include the relevant contributions from the fermions.

5. Gauge theory

Let us first consider the light-cone hamiltonian

$$H = i\partial_u = \frac{\partial\tau}{\partial u}i\partial_\tau + \frac{\partial\phi}{\partial u}i\partial_\phi + \frac{\partial\beta}{\partial u}i\partial_\beta + \frac{\partial\gamma}{\partial u}i\partial_\gamma\,, \tag{5.1}$$

where

$$\begin{aligned} \tau &= Eu - \frac{1}{2\Omega_0^2}\frac{v}{EL^2}\,, \qquad \psi = \frac{3}{2}\left(Eu + \frac{1}{2\Omega_0^2}\frac{v}{EL^2}\right), \\ \phi &= \hat{\phi} + \frac{1}{3}\psi\,, \qquad \gamma = \hat{\gamma} + \frac{2}{3}\psi\,. \end{aligned} \tag{5.2}$$

Since $\mathrm{U}(1)_\beta$ is not a symmetry of the gauge theory superpotential, there is no conserved charge associated with the operator $i\partial_\beta$, and so it would not make sense to have this term present in the hamiltonian. Happily, however, we have

$$\beta = \psi - (\phi + \gamma) = -\left(\hat{\phi} + \hat{\gamma}\right), \tag{5.3}$$

so that $\partial\beta/\partial u = 0$ as required. The scaling dimension, Δ, the R-charge, J, and the "flavour" charge, J_3, associated with the $\mathrm{U}(1)_\gamma$ diagonal subgroup of the global "flavour" SU(2), are given by

$$\Delta = i\partial_\tau\,, \qquad J = -i\partial_\phi\,, \qquad J_3 = i\partial_\gamma\,, \tag{5.4}$$

so that, setting $E = 1$ for convenience,

$$H = \Delta - \frac{1}{2}J + J_3\,. \tag{5.5}$$

Likewise, the light-cone momentum is given by

$$P = i\partial_v = -\frac{1}{2\Omega_0^2}\frac{1}{L^2}\left(\Delta + \frac{J}{2} - J_3\right). \tag{5.6}$$

Since both of these quantities should remain fixed after taking the Penrose limit, in analogy with ref. [21], we are interested in operators with large R- and flavour-charges:

$$J, J_3 \sim L^2 \sim N^{1/2}\,, \tag{5.7}$$

as we take the $N \to \infty$ limit, keeping g_{YM}^2 fixed and small. In this limit of *infinite* 't Hooft coupling, we must further demand that $\Delta - (J/2) + J_3$ is kept fixed, so that the light-cone hamiltonian remains finite.

The values of Δ, J, J_3 and H for the complex scalar fields appearing as the lowest-order components in the expansion of the three chiral and three anti-chiral superfields are listed in table 1. Remembering that it is Φ_3 which is massive, and can be integrated out as $\Phi_3 \sim [\Phi_1, \Phi_2]$, the values of H which we find make sense: the energy of ϕ_3 is equal to the sum of the energies of ϕ_1 and ϕ_2.

The first prediction from the spectrum found in section 4 is that there should be a *unique* light-cone ground state with large Δ, J and J_3. It is simply that state for which all the occupation numbers in (4.48) vanish. This corresponds in the gauge theory to the operator $\mathrm{Tr}(\phi_2^{2J})$. It has $H = 0$ since it is chiral — its conformal dimension is simply the naïve value $\Delta = 3J/2$. The second prediction (and this is where we depart from the previous results concerning both the $\mathcal{N} = 4$ [21] and $\mathcal{N} = 1$ [27, 28, 29] theories) is that there should be precisely *six* bosonic operators with $H = 1$ and *two* bosonic operators with $H = 2$, corresponding to the zero modes of the worldsheet scalars. Four are straightforward to write down; they are simply derivatives of the ground state operator

$$\mathrm{Tr}\left(D_k\phi_2^{2J}\right), \qquad (k = 1, 2, 3, 4)\,, \tag{5.8}$$

coming from inserting covariant derivatives along any of the four spacetime directions (in an Euclidean discussion). This follows straightforwardly from the descendant of the action of the conformal group.

	Δ	J	J_3	H
ϕ_1	$3/4$	$1/2$	$1/2$	1
ϕ_2	$3/4$	$1/2$	$-1/2$	0
ϕ_3	$3/2$	1	0	1
$\bar{\phi}_1$	$3/4$	$-1/2$	$-1/2$	$1/2$
$\bar{\phi}_2$	$3/4$	$-1/2$	$1/2$	$3/2$
$\bar{\phi}_3$	$3/2$	-1	0	2

Table 1: The conformal dimensions, charges and light-cone energies of the complex scalar fields appearing as the lowest-order components in the expansions of the three chiral and three anti-chiral superfields.

Another operator with $H = 1$ is $\mathrm{Tr}(\phi_1 \phi_2^{2J})$. This is again chiral and so its conformal dimension is the sum of those of its constituents. This leaves a sixth bosonic operator to be found, which we can look for in analogy with the analysis of Itzhaki et al. [27] in the $T^{1,1}$ case. We propose

$$\mathrm{Tr}\left(\bar{\phi}_1 \phi_2^{2J}\right), \tag{5.9}$$

as our sixth operator with $H = 1$. Since it is not chiral, its conformal dimension is not necessarily the naïve one found using the values in table 1.

To understand why this operator should have $\Delta = (6J + 5)/4$, consider

$$\mathrm{Tr}\left(\bar{\phi}_1 \phi_2\right). \tag{5.10}$$

Perusal of the tables in ref. [13], shows that this operator is in the same $\mathcal{N} = 2$ supermultiplet as the conserved SU(2) current and therefore its dimension is the same as its free-field value, i.e. $\Delta = 2$, which gives $H = 1$. Unfortunately there does not seem to be a field theory method to derive the conformal dimension of $\mathrm{Tr}(\bar{\phi}_1 \phi_2^{2J})$. For the equivalent operator in the $T^{1,1}$ case, Itzhaki et al. [27] were able to find the relevant conformal dimension using a standard AdS/CFT formula relating the conformal dimension to the laplacian on $T^{1,1}$. In the case of the Pilch-Warner geometry considered herein, we do not have such a formula and so the proposed conformal dimension of $\mathrm{Tr}(\bar{\phi}_1 \phi_2^{2J})$ is somewhat more conjectural than one might have liked.

We should also consider the two operators with $H = 2$, which contribute at the same "level" as the six operators with $H = 1$. However, there are a host of candidates, and none of them seem to have protected conformal dimensions. Indeed, partly for this reason, we have not been able to satisfactorily identify these operators, and so leave it as a conjecture that there are precisely two such operators with $H = 2$ and large Δ, J and J_3 dual to those seen in the string theory spectrum.

Given this limited success with the bosonic operators at the lowest lying levels, let us now turn our attention to the fermionic ones. From table 2 one can immediately see that

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	Δ	J	J_3	H
χ_1	5/4	$-1/2$	1/2	2
χ_2	5/4	$-1/2$	$-1/2$	1
ψ	3/2	1	0	1
$\bar{\chi}_1$	5/4	1/2	$-1/2$	1/2
$\bar{\chi}_2$	5/4	1/2	1/2	3/2
$\bar{\psi}$	3/2	-1	0	2

Table 2: The conformal dimensions, charges and light-cone energies of the gauginos, ψ, and the fermionic components, χ_1 and χ_2, of the chiral superfields Φ_1 and Φ_2, and their anti-chiral counterparts. We do not consider the components of Φ_3 here, since it should not enter our discussion at all.

the following operators have $H = 1$:

$$\mathrm{Tr}\left(\chi_2 \phi_2^{2J}\right), \qquad \mathrm{Tr}\left(\psi \phi_2^{2J}\right). \tag{5.11}$$

These give four fermionic operators since both χ and ψ are two-component Weyl fermions. The first two are the supersymmetry variation of the ground state operator. The second two involve the gaugino, ψ. The remaining two operators to be found are the fermionic counterparts of $\mathrm{Tr}(\bar{\phi}_1\phi_2)$:

$$\mathrm{Tr}\left(\bar{\chi}_1 \phi_2^{2J}\right). \tag{5.12}$$

Again, we do not consider fermionic operators with $H = 2$, but expect that there are precisely two of them as above.

Of course, our ultimate aim should be to reproduce the form of the string spectrum (4.48) from the gauge theory, along the lines of Berenstein, Maldacena and Nastase [21]. Rewriting this in gauge theory variables, we would want to derive that, for the $5, 6, 7, 8$ directions,

$$\left(\Delta - \hat{J}\right)_n = \sqrt{1 + \frac{cn^2 g_{\mathrm{YM}}^2 N}{\hat{J}^2}}, \tag{5.13}$$

where $\hat{J} = J/2 - J_3$, c is the ratio of central charges [46, 13] $c_{\mathrm{IR}}/c_{\mathrm{UV}} = 27/32$, and we have used the fact that $L^4 = g_{\mathrm{YM}}^2 N \alpha'^2$ and $P \sim \hat{J}/L^2$. For the $1, 2, 3, 4$ directions, we should have the very interesting result

$$\left(\Delta - \hat{J}\right)_n = \sqrt{\frac{5}{2} + \frac{cn^2 g_{\mathrm{YM}}^2 N}{\hat{J}^2} \pm \frac{1}{2}\sqrt{9 + 12\frac{cn^2 g_{\mathrm{YM}}^2 N}{\hat{J}^2}}}. \tag{5.14}$$

We will leave the task of verifying this prediction directly from gauge theory for future work. See section 1 for more discussion of the gauge theory implications.

Having considered the IR fixed point geometry and its Penrose limits in some detail, we now move away from the fixed point, turning to the flow geometry of ref. [39].

6. Taking the Penrose limit along the flow

If we are to take the Penrose limit of the flow geometry away from the fixed points, we are forced to use "Poincare" coordinates on the "AdS" space. Using the the same coordinates on the squashed five-sphere as in section 3 above, the flow geometry is described by the metric [39]

$$\begin{aligned} \mathrm{d}s^2 &= \frac{X^{1/2}\cosh\chi}{\rho}\left(e^{2A}\mathrm{d}s^2(\mathbb{M}^4)+\mathrm{d}r^2\right)+\mathrm{d}s_5^2\,, \\ \mathrm{d}s_5^2 &= L_0^2\frac{X^{1/2}\,\mathrm{sech}\,\chi}{\rho^3}\Bigg[\mathrm{d}\theta^2+\frac{\rho^6\cos^2\theta}{X}\left(\sigma_1^2+\sigma_2^2\right)+\frac{\rho^{12}\sin^2(2\theta)}{4X^2}\left(\sigma_3+\frac{(2+\rho^6)}{2\rho^6}\mathrm{d}\phi\right)^2+ \\ &\qquad +\frac{\rho^6\cosh^2\chi}{4X^2}\left(2\sin^2\theta-\cos^2\theta\right)^2\left(\mathrm{d}\phi-\frac{2\cos^2\theta}{(2\sin^2\theta-\cos^2\theta)}\sigma_3\right)^2\Bigg], \end{aligned} \tag{6.1}$$

where

$$X=\cos^2\theta+\rho^6\sin^2\theta\,. \tag{6.2}$$

We are still free to choose geodesics for which $\theta=0$ and $\alpha=0$. Considering also a constant point on $\mathbb{E}^3$, the effective lagrangian is

$$\mathcal{L}=\frac{\cosh\chi}{\rho}\left(-e^{2A}\dot{t}^2+\dot{r}^2+\frac{L_0^2}{4}\rho^4\dot{\psi}^2\right), \tag{6.3}$$

where ψ is defined in (3.9). As above, there are two conserved quantities, E and h, associated with the Killing vectors $\partial/\partial t$ and $\partial/\partial\psi$, respectively. The t and ψ equations then give

$$\dot{t}=EL_0\frac{\rho}{\cosh\chi}e^{-2A}\,,\qquad \dot{\psi}=\frac{h}{\rho^3\cosh\chi}\,, \tag{6.4}$$

and the null condition is

$$\dot{r}=EL_0\frac{\rho}{\cosh\chi}\sqrt{e^{-2A}-\frac{h^2}{E^2}\frac{1}{4\rho^4}}\,, \tag{6.5}$$

where we have chosen the arbitrary sign in the above to be positive. Of course, we cannot integrate to find $r(\lambda)$, but we do not need to.

Following refs. [20, 22], we introduce coordinates $\{u,v,x\}$ such that $g_{uu}=0=g_{ux}$ and $g_{uv}=1$. In other words, just as in (D.10), we have

$$\begin{aligned} \partial_u &= \dot{r}\partial_r+\dot{t}\partial_t+\dot{\psi}\partial_\psi\,, \\ \partial_v &= -\frac{1}{EL_0}\partial_t\,, \\ \partial_x &= \frac{1}{L_0}\partial_\psi+\frac{1}{4}\frac{h}{E}\partial_t\,, \end{aligned} \tag{6.6}$$

which gives

$$\mathrm{d}r=EL_0\frac{\rho}{\cosh\chi}\left(e^{-2A}-\frac{h^2}{E^2}\frac{1}{4\rho^4}\right)^{1/2}\mathrm{d}u\,,$$

$$\begin{aligned}
\mathrm{d}t &= EL_0\frac{\rho}{\cosh\chi}e^{-2A}\mathrm{d}u - \frac{\mathrm{d}v}{EL_0} + \frac{1}{4}\frac{h}{E}\mathrm{d}x\,,\\
\mathrm{d}\psi &= \frac{h}{\rho^3\cosh\chi}\mathrm{d}u + \frac{\mathrm{d}x}{L_0}\,.
\end{aligned} \tag{6.7}$$

Substituting for these in the metric (6.1), taking

$$\theta = \frac{y}{L_0}\,, \qquad \alpha = \frac{w}{L_0}\,, \tag{6.8}$$

and dropping all terms of $\mathcal{O}(1/L_0)$, we find

$$\begin{aligned}
\mathrm{d}s^2 &= 2\mathrm{d}u\mathrm{d}v + \frac{1}{4}\rho^3\cosh\chi e^{2A}\left(e^{-2A} - \frac{h^2}{E^2}\frac{1}{4\rho^4}\right)\mathrm{d}x^2 + \frac{\cosh\chi}{\rho}\,e^{2A}\mathrm{d}s^2(\mathbb{E}^3) + \\
&\quad + \frac{\operatorname{sech}\chi}{\rho^3}\left(\mathrm{d}y^2 + y^2\mathrm{d}\hat{\phi}^2\right) + \frac{1}{4}\operatorname{sech}\chi\rho^3\left(\mathrm{d}w^2 + w^2\mathrm{d}\hat{\gamma}^2\right) - \frac{h^2}{4}\cosh\chi\left(\rho^3 y^2 + \frac{1}{4}\frac{w^2}{\rho^3}\right)\mathrm{d}u^2\,,
\end{aligned} \tag{6.9}$$

where

$$\mathrm{d}\hat{\phi} = \mathrm{d}\phi - \rho^6\sinh^2\chi\frac{\mathrm{d}\psi}{2}\,, \qquad \mathrm{d}\hat{\gamma} = \mathrm{d}\gamma - \cosh^2\chi\frac{\mathrm{d}\psi}{2}\,. \tag{6.10}$$

Note that in the IR, these reduce to the angular variables in (3.12) as required.

To write this in terms of Brinkman coordinates, define

$$\begin{aligned}
E(u) &= \frac{1}{2}\rho^{3/2}\cosh^{1/2}\chi e^{A}\sqrt{e^{-2A} - \frac{h^2}{E^2}\frac{1}{4\rho^4}} = \frac{1}{2EL_0}\rho^{1/2}\cosh^{3/2}\chi\; e^{A}\dot{r}\,,\\
F(u) &= e^{A}\sqrt{\frac{\cosh\chi}{\rho}}\,, \qquad G(u) = \sqrt{\frac{\operatorname{sech}\chi}{\rho^3}}\,, \qquad H(u) = \frac{1}{2}\sqrt{\rho^3\operatorname{sech}\chi}\,,
\end{aligned} \tag{6.11}$$

and consider the metric

$$\begin{aligned}
\mathrm{d}s^2 &= 2\mathrm{d}u\mathrm{d}v + E(u)^2\mathrm{d}x^2 + F(u)^2\mathrm{d}x^i\mathrm{d}x^i + G(u)^2\mathrm{d}z^1\mathrm{d}\bar{z}^1 + \\
&\quad + H(u)^2\mathrm{d}z^2\mathrm{d}\bar{z}^2 - \frac{h^2}{4}\cosh\chi\left(\rho^3|z^1|^2 + \frac{|z^2|^2}{4}\right)\mathrm{d}u^2,
\end{aligned} \tag{6.12}$$

where $i = 1,2,3$ and z^1, z^2 are complex coordinates on the obvious $\mathbb{E}^2$s. Then, with a dot denoting $\partial/\partial u$, the relevant Brinkman coordinates are

$$\begin{aligned}
\hat{u} &= u\,, \qquad \hat{x} = Ex\,, \qquad \hat{x}^i = Fx^i\,, \qquad \hat{z}^1 = Gz^1\,, \qquad \hat{z}^2 = Hz^2\,,\\
\hat{v} &= v - \frac{1}{2}\left(E\dot{E}x^2 + F\dot{F}x^ix^i + G\dot{G}|z^1|^2 + H\dot{H}|z^2|^2\right),
\end{aligned} \tag{6.13}$$

in terms of which the metric (6.12) becomes, dropping the hats,

$$\begin{aligned}
\mathrm{d}s^2 &= 2\mathrm{d}u\mathrm{d}v + \mathrm{d}s^2(\mathbb{E}^8) - \left[-\frac{\ddot{E}}{E}x^2 - \frac{\ddot{F}}{F}x^ix^i + \right.\\
&\quad \left. + \left(\frac{h^2}{4}\rho^3 - \frac{\ddot{G}}{G}\right)|z^1|^2 + \left(\frac{h^2}{16}\frac{1}{\rho^3} - \frac{\ddot{H}}{H}\right)|z^2|^2\right]\mathrm{d}u^2\,.
\end{aligned} \tag{6.14}$$

We will not consider the form fields explicitly, but it is easy to see that an application of the Penrose limit will give the same fields (3.14) as for the IR solution of section 3, but with a u-dependent amplitude.

At any rate, the resulting metric is certainly in the form of a one-half supersymmetric pp-wave, but with a complicated u-dependent profile. It seems unlikely that string theory on this background is tractable. Moreover, it is somewhat difficult to see what statements about the dual gauge theory can be made. The immediate observation in this regard is, of course, that there is no concept of operators with a definite conformal dimension at a general point along the flow. However, in the maximally supersymmetric case, dual to the $\mathcal{N} = 4$ Yang-Mills theory, we know [40, 58, 59] that evolution in light-cone time u corresponds to changes of scale in the gauge theory (the original holographic radial direction is a monotonic function of u). It is thus tempting to argue that string theory on the above pp-wave is dual to an "RG flow" between the Penrose limit of the $\mathcal{N} = 4$ Yang-Mills theory ($u = \infty$) and the Penrose limit of the $\mathcal{N} = 1$ fixed point theory ($u = -\infty$). Evolution in light-cone time would then induce a flow between the relevant sectors of the two gauge theories.

However, the interpretation must be more subtle as is apparent from considering the Penrose limit of $AdS_5 \times S^5$ in Poincaré coordinates. For example in (D.8), one finds that the usual null trajectories start at $r = 0$, travel out to some maximum $r = L\ln(E/h)$ and then fall back to $r = 0$. Hence these geodesics sample a finite range of energies extending from the far IR to some maximum, which depends solely on the choice of the initial conditions for the geodesic. Clearly, unravelling information about the RG flow with the analogous geodesics above is a challenging but potentially fruitful problem. We should also add that similar geodesics in nonconformal backgrounds were considered in refs. [60, 61], and a discussion of RG flows in this context also appeared in ref. [58].

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A. The Pilch-Warner geometry

The ten-dimensional Pilch-Warner geometry has the metric [35, 39]

$$\mathrm{d}s^2 = \Omega^2 \left(e^{2A}\mathrm{d}s^2(\mathbb{M}^4) + \mathrm{d}r^2\right) + \mathrm{d}s_5^2\,, \qquad \text{(A.1)}$$

where, in terms of Cartesian coordinates $x^I, I = 1, \ldots, 6$, on $\mathbb{E}^6$ such that $x^I x^I = 1$, the five-dimensional "internal" metric is

$$\mathrm{d}s_5^2 = L_0^2 \frac{\operatorname{sech}\chi}{\xi^3} \left[\xi^2 \mathrm{d}x^I Q^{-1}_{IJ} \mathrm{d}x^J + \sinh^2\chi \left(x^I J_{IJ} \mathrm{d}x^J\right)^2\right], \tag{A.2}$$

L_0 being the radius of the AdS space at the UV fixed point. The complex structure $J_{IJ} = -J_{JI}$ has non-zero components $J_{14} = J_{23} = J_{65} = 1$ and

$$\Omega^2 = \xi \cosh\chi\,, \qquad \xi^2 = x^I Q_{IJ} x^J\,, \qquad Q = \operatorname{diag}\left(\rho^{-2}, \rho^{-2}, \rho^{-2}, \rho^{-2}, \rho^4, \rho^4\right). \tag{A.3}$$

The supergravity scalars $\chi(r)$ and $\rho(r)$ obey, together with the metric function $A(r)$, the following equations [13]:

$$\begin{aligned}
\frac{\mathrm{d}\rho}{\mathrm{d}r} &= \frac{1}{6L_0\rho}\left(\cosh(2\chi)(\rho^6+1) - (3\rho^6-1)\right),\\
\frac{\mathrm{d}\chi}{\mathrm{d}r} &= \frac{1}{2L_0\rho^2}\sinh(2\chi)\left(\rho^6-2\right),\\
\frac{\mathrm{d}A}{\mathrm{d}r} &= -\frac{1}{6L_0\rho^2}\left(\cosh(2\chi)(\rho^6-2) - (3\rho^6+2)\right).
\end{aligned} \tag{A.4}$$

As explained in ref. [35], one uses the complex coordinates

$$u^1 = x^1 + ix^4\,, \qquad u^2 = x^2 + ix^3\,, \qquad u^3 = x^5 - ix^6\,, \tag{A.5}$$

parametrised as

$$\begin{pmatrix} u^1 \\ u^2 \end{pmatrix} = e^{-i\phi/2}\cos\theta\, g \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad u^3 = e^{-i\phi}\sin\theta\,, \tag{A.6}$$

where

$$g = \begin{pmatrix} v^1 & -\bar{v}^2 \\ v^2 & \bar{v}^1 \end{pmatrix} \in \mathrm{SU}(2)\,, \tag{A.7}$$

is used to define the SU(2) left-invariant one-forms, σ_i, in terms of the Pauli matrices, τ_i, via

$$\sigma_i = -\frac{i}{2}\operatorname{Tr}\left(\tau_i g^{-1}\mathrm{d}g\right). \tag{A.8}$$

Our choice of these one-forms differs from that of refs. [35, 39] by a factor of 1/2, which gives rise to various discrepancies between the form-fields given here and those in refs. [35, 39]. Our left-invariant one-forms satisfy $\mathrm{d}\sigma_i = \varepsilon_{ijk}\sigma_j \wedge \sigma_k$ so the metric on the unit 3-sphere is $\mathrm{d}\Omega_3^2 = \sigma_i\sigma_i$. In terms of Euler angles $\{\alpha, \beta, \gamma\}$ on the three-sphere, we have

$$v^1 = \cos\left(\frac{\alpha}{2}\right) e^{i(\beta+\gamma)/2}, \qquad v^2 = \sin\left(\frac{\alpha}{2}\right) e^{i(\beta-\gamma)/2}\,, \tag{A.9}$$

so that

$$\begin{aligned}
\sigma_1 &= \frac{1}{2}\left(\sin\beta\,\mathrm{d}\alpha - \cos\beta\sin\alpha\,\mathrm{d}\gamma\right),\\
\sigma_2 &= -\frac{1}{2}\left(\cos\beta\,\mathrm{d}\alpha + \sin\beta\sin\alpha\,\mathrm{d}\gamma\right),\\
\sigma_3 &= \frac{1}{2}\left(\mathrm{d}\beta + \cos\alpha\,\mathrm{d}\gamma\right).
\end{aligned} \tag{A.10}$$

There is thus a natural U(1) action $\beta \to \beta +$ constant, under which the SU(2) doublet in (A.6) picks up an overall phase. This global $\mathrm{U}(1)_\beta$ rotates σ_1 into σ_2 and leaves σ_3 invariant. The local version can be used to choose different parametrisation of the five-sphere directions. Thus, to go from the coordinates used in ref. [35] to those used in ref. [39], one shifts $\beta \to \beta + \phi$, which removes the overall phase from the SU(2) doublet in (A.6), and induces the shift $\sigma_3 \to \sigma_3 + \phi/2$. There is a further U(1) action $\gamma \to \gamma +$ constant which will be of interest to us. This $\mathrm{U}(1)_\gamma$ is the diagonal subgroup of the obvious global SU(2). Roughly speaking, the doublet corresponds to the two massless chiral superfields, Φ_1 and Φ_2, in the gauge theory, and the singlet corresponds to the massive superfield Φ_3. More precisely [49, 50], the moduli space of a D3-brane probe corresponds to the former, $\theta = 0$, directions and the latter, $\theta = 0$, directions are orthogonal to this moduli space.

Putting this all together, we have (cf. ref. [35])

$$\begin{aligned} \mathrm{d}s^2 &= \frac{X^{1/2}\cosh\chi}{\rho}\left(e^{2A}\mathrm{d}s^2(\mathbb{M}^4) + \mathrm{d}r^2\right) + \mathrm{d}s_5^2\,, && \text{(A.11)}\\ \mathrm{d}s_5^2 &= L_0^2\frac{X^{1/2}\,\mathrm{sech}\,\chi}{\rho^3}\left[\mathrm{d}\theta^2 + \frac{\rho^6\cos^2\theta}{X}\left(\sigma_1^2+\sigma_2^2\right) + \frac{\rho^{12}\sin^2(2\theta)}{4X^2}\left(\sigma_3 + \frac{(2-\rho^6)}{2\rho^6}\mathrm{d}\phi\right)^2 + \right.\\ &\qquad \left. + \frac{\rho^6\cosh^2\chi}{16X^2}(3-\cos(2\theta))^2\left(\mathrm{d}\phi - \frac{4\cos^2\theta}{(3-\cos(2\theta))}\sigma_3\right)^2\right]\,, && \text{(A.12)} \end{aligned}$$

where

$$X(r,\theta) = \cos^2\theta + \rho^6\sin^2\theta\,. \tag{A.13}$$

Note that the global isometry group of the metric is $\mathrm{SU}(2)\times\mathrm{U}(1)_\beta\times\mathrm{U}(1)_\phi$, although only a combination of the two U(1)s is preserved by the form-fields. In the IR ($r \to -\infty$), we have $\chi \to 2/\sqrt{3}$, $\rho \to 2^{1/6}$ and $A(r) \to r/L$, where $L = (3/2^{5/3})L_0$. The metric becomes [35]

$$\begin{aligned} \mathrm{d}s^2(\mathrm{IR}) &= \frac{2^{1/3}}{\sqrt{3}}(3-\cos 2\theta)^{1/2}\left(e^{2r/L}\mathrm{d}s^2(\mathbb{M}^4) + \mathrm{d}r^2\right) + \mathrm{d}s_5^2(\mathrm{IR})\,,\\ \mathrm{d}s_5^2(\mathrm{IR}) &= \frac{\sqrt{3}L_0^2}{4}(3-\cos 2\theta)^{1/2}\left[\mathrm{d}\theta^2 + \frac{4\cos^2\theta}{(3-\cos 2\theta)}(\sigma_1^2+\sigma_2^2) + \frac{4\sin^2 2\theta}{(3-\cos 2\theta)^2}\sigma_3^2 + \right.\\ &\qquad \left. + \frac{2}{3}\left(\mathrm{d}\phi - \frac{4\cos^2\theta}{(3-\cos 2\theta)}\sigma_3\right)^2\right]\,. \end{aligned} \tag{A.14}$$

Concentrating for the time being on this fixed point geometry, our self-dual five-form is

$$F_5(\mathrm{IR}) = -\frac{2}{3}\frac{2^{2/3}}{L}e^{4r/L}\,(1+\star)\,\epsilon(\mathbb{E}^4)\wedge\mathrm{d}r\,, \tag{A.15}$$

which differs by a factor of 2 to that in ref. [35]. To determine the correct Ansatz for the three-form, one considers the linear $G_3 = \mathrm{d}u^1\wedge\mathrm{d}u^2\wedge\mathrm{d}u^3$, which depends on ϕ only through the overall phase $e^{-2i\phi}$, and which includes an overall factor of $(\sigma_1 + i\sigma_2)$ (this is $(\sigma_1 - i\sigma_2)$ in ref. [35, 39] due to different conventions for the left-invariant one-forms). The two-form potential is thus

$$A_2(\mathrm{IR}) = Ae^{-2i\phi}\frac{L_0^2\cos\theta}{2}\left(\mathrm{d}\theta - \frac{2i\sin(2\theta)}{(3-\cos(2\theta))}\sigma_3\right)\wedge(\sigma_1 + i\sigma_2)\,, \tag{A.16}$$

where there is an overall arbitrary constant phase, A which is set to $-i$ in ref. [35]. The field strength $G_3 = \mathrm{d}A_2$ is

$$\begin{aligned} G_3(\mathrm{IR}) &= iA\; e^{-2i\phi} L_0^2 \cos\theta\, \times \\ &\times \left(\mathrm{d}\theta\wedge\mathrm{d}\phi - \frac{8\cos^2\theta}{(3-\cos(2\theta))^2}\mathrm{d}\theta\wedge\sigma_3 - \frac{2i\sin(2\theta)}{(3-\cos(2\theta))}\sigma_3\wedge\mathrm{d}\phi \right)\wedge(\sigma_1+i\sigma_2)\,. \end{aligned} \tag{A.17}$$

It should be obvious that the global U(1) symmetry group of the solution as a whole is the combination $\mathrm{U}(1)_R = \mathrm{U}(1)_\phi + 2\,\mathrm{U}(1)_\beta$. Of course, by shifting β as discussed above, one is free to choose the R-symmetry to be any combination of the two U(1)s. For example, in the text we are interested in coordinates for which $\mathrm{U}(1)_R = \mathrm{U}(1)_\phi$, so we perform the coordinate transformation $\beta \to \beta + 2\phi$ on the above solution. This removes the overall ϕ-dependent phase in the two-form potential precisely as required.

B. The $\theta = \pi/2$ geodesics

Here we will consider the Penrose limit of the IR fixed point solution along a null geodesic with $\theta = \pi/2$, corresponding to the massive direction orthogonal to the moduli space. We start with the solution (3.1)–(3.4), and take

$$\rho = \frac{r}{L}\,, \qquad \theta = \frac{\pi}{2} + \frac{y}{L}\,, \tag{B.1}$$

with $L\to\infty$. Defining the light-cone coordinates,

$$u = \frac{1}{2E}\left(\tau + \frac{2}{3}\phi\right), \qquad v = -\frac{2^{4/3}}{\sqrt{3}}EL^2\left(\tau - \frac{2}{3}\phi\right), \tag{B.2}$$

the solution becomes

$$\begin{aligned} \mathrm{d}s_2^2 &= 2\mathrm{d}u\mathrm{d}v + \frac{E^2}{4}\left(1-4(r^2+|v|^2)\right)\mathrm{d}u^2 + \mathrm{d}s^2\left(\mathbb{E}^8\right) + iE\mathrm{d}u\left(v^1\mathrm{d}\bar{v}^1 + v^2\mathrm{d}\bar{v}^2\right), \\ F_5 &= -\frac{E}{2}\mathrm{d}u\wedge\epsilon\left(\mathbb{E}^4\right), \\ G_3 &= -\sqrt{3}E\mathrm{d}u\wedge\mathrm{d}v^1\wedge\mathrm{d}v^2\,, \end{aligned} \tag{B.3}$$

where we have rescaled r and y and defined the coordinates

$$v^1 = e^{-iEu/2}z^1\,, \qquad v^2 = e^{-iEu/2}z^2\,, \tag{B.4}$$

z^1, z^2 being complex coordinates on the two $\mathbb{E}^4$s. The constant in g_{uu} is unimportant as far as the field equations and supersymmetry transformations are concerned, and has been discussed in ref. [30]. The metric has a $\mathrm{d}u\mathrm{d}x^i$ cross-term, but it is clear that this can be traded with explicit u-dependence in the three-form.

In the original coordinates, that is, the cross term is of the form $\sigma_3\mathrm{d}u$ and can be removed by shifting the Euler angle β, to give

$$\begin{aligned} \mathrm{d}s_2^2 &= 2\mathrm{d}u\mathrm{d}v - E^2(r^2+|z|^2)\mathrm{d}u^2 + \mathrm{d}s^2\left(\mathbb{E}^8\right), \\ F_5 &= -\frac{E}{2}\mathrm{d}u\wedge\epsilon\left(\mathbb{E}^4\right), \\ G_3 &= -\sqrt{3}Ee^{-iEu}\mathrm{d}u\wedge\mathrm{d}z^1\wedge\mathrm{d}z^2\,. \end{aligned} \tag{B.5}$$

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Substituting for $\lambda = -E/2$ and $\mu = -\sqrt{3}Ee^{-iEu}$ in the field eq. (2.11), one can verify that the solution is valid. The u-dependence in G_3 drops out of the field equations since it is just an overall phase. We note that the above pp-wave will give rise to worldsheet scalars of the same mass, unlike the case we have considered in the text.

We have not been able to understand the significance of this particular Penrose limit with respect to the dual gauge theory. In the original coordinates with a cross term in the metric, the light-cone hamiltonian one finds is

$$H = \Delta - \frac{3}{2}J\,, \tag{B.6}$$

so that all six scalar fields have $H = 0$ — there is certainly no unique ground state. Moreover, this fact does not seem to be mirrored in the string theory spectrum on this background, which *does* seem to show a unique ground state. Furthermore, the frequencies of the bosonic and fermionic modes do not seem to match, in which case it seems unlikely that a simple hamiltonian can be written down at all. On the other hand, after shifting β to remove the cross-term, one finds an $i\partial_\beta$ term in the hamiltonian and, as discussed in section 5, there is no conserved charge in the gauge theory associated with this differential operator.

Taking the Penrose limit along geodesics with angular momentum in the massive directions is perhaps an odd thing to try to do anyway, since at the IR fixed point, one can simply integrate out these directions. As far as a D-brane probe would be concerned, motion in these directions is energetically disfavoured and simply not to be described in the dual picture by the effective low energy $\mathcal{N} = 1$ field theory.

C. Instabilities for large B-fields

Let us consider the general solution (2.10), with a diagonal matrix $A_{ij} = -\delta_{ij}E_i^2$, for which the supergravity equations of motion (2.11) yield:

$$\sum E_i^2 = 8\mu^2 + 2|\zeta|^2\,. \tag{C.1}$$

Following the analysis of section 4.1, the equations of motion for X^1 and X^3 are then

$$\nabla^2 X^1 - M_1^2 X^1 + b\partial_\sigma X^3 = 0\,, \tag{C.2}$$
$$\nabla^2 X^3 - M_3^2 X^3 - b\partial_\sigma X^1 = 0\,, \tag{C.3}$$

where $b = p^+\alpha'\zeta$ and $M_i = p^+\alpha' E_i$. (Similar equations hold for the X^2 and X^4 directions.) Fourier expanding as in (4.10) above, one finds that the frequencies of the normal modes are:

$$\omega_n^2 = n^2 + \frac{M_1^2 + M_3^2}{2} \pm \sqrt{\frac{1}{4}(M_1^2 - M_3^2)^2 + b^2 n^2}\,. \tag{C.4}$$

Now note that *all* of the above ω_n's are real and non-zero if and only if

$$b^2 < n^2 + M_1^2 + M_3^2 + \frac{M_1^2 M_3^2}{n^2}\,. \tag{C.5}$$

In particular, by minimising the right-hand side with respect to n, one is guaranteed real ω_n for

$$b^2 < (M_1 + M_3)^2 \,. \tag{C.6}$$

However, for larger values of b^2, it *is* possible that some of the frequencies are imaginary, resulting in exponentially growing string modes.

Generalising the analysis of section 4.2, one finds no such instability in the fermionic spectrum. As well as the standard four fermions with frequency $\omega_n^2 = n^2 + M^2$ at each level, the remaining fermionic oscillators have

$$\omega_n^2 = n^2 + M^2 + \frac{b^2}{2} \pm \frac{1}{2}\sqrt{b^4 + 4b^2n^2} \,, \tag{C.7}$$

where $M = p^+\alpha'\mu$. It is straightforward to show that this expression always yields real frequencies. Note that the metric coefficients E_i^2 do not appear directly in the fermionic spectrum. Further, for the general background, the bosonic and fermionic spectra no longer match.

The interpretation of the unstable modes is somewhat unclear, although their existence is quite interesting. One might think of them as some sort of (classical) instability in the string theory in these backgrounds. The appearance of these modes is particularly curious because the supergravity background still appears to be at least one-half supersymmetric, i.e. the 16 standard Killing spinors annihilated by Γ_+ will yield vanishing supersymmetry variations (2.4), irrespective of the value of the three-form field. One might suspect that these Killing spinors are ill-behaved in some way, e.g. exponentially diverging in u, if the three-form is too large. However, this is not the case as is obvious since as noted above the equation for standard Killing spinors is independent of G_3.

One might imagine that solutions with the B-field too large, in the above sense, are excluded by the supergravity field eqs. (C.1). However, it is easy to see that this is not the case, as the inequality (C.6) only refers to three of the nine parameters appearing in the former equation. Hence these unstable modes apparently appear in valid supergravity backgrounds. Further it seems that given the null form of these gravity wave solutions, they will be solutions of the string equations of motion to all orders in α' [51, 52, 53]. In particular, it seems the general discussion of ref. [52] should apply even with the appearance of R-R fields in the background.

Let us make several further observations. First, these solutions are in no sense asymptotically flat in any directions, rather the field strengths and R_{uiuj} are constant throughout the spacetime. Hence one may wonder whether or not the generic backgrounds are relevant in string theory. Certainly we have found that certain pp-wave solutions (in fact a very broad family, given the results of section 6 and appendix B) appear as the Penrose limits in asymptotically AdS backgrounds, and so play a role in string theory. It could be that these "unstable" supergravity solutions are simply pathological backgrounds as far as the string theory is concerned and are not useful spacetimes to consider from this point of view. A second observation is that the unstable modes in the bosonic spectrum only occur at finite, non-zero n. Roughly, one may think that oscillator modes need to be excited so that the string is spatially extended and can "see" the B-field. Hence the instability

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is inherently stringy in origin. This feature is somewhat reminiscent of the instabilities discussed in ref. [62]. Finally, we note that the instability only appears for a finite set of modes, i.e. for a finite range of n^2. It is straightforward to derive the exact range, however, let us make some qualitative statements. Generically if we take $M_1^2 \simeq M_3^2 \simeq M^2$ then the instability sets in for $b^2 \gtrsim M^2$. In this case the unstable modes appear in a certain range, $n_-^2 < n^2 < n_+^2$, where $n_\pm^2 = \mathcal{O}(M^2)$ and $n_+^2 - n_-^2 = \mathcal{O}(M^2)$. However, recall the definitions above, $M_i^2 = (p^+\alpha' E_i)^2$. Now in studying supergravity backgrounds, we would ask that typical curvatures are small which in this case corresponds to $(l_s E_i)^2 \ll 1$. If this inequality applies and $(l_s p^+)^2 \lesssim 1$, then the unstable range will lie entirely within the range 0 and 1. That is, there will not actually be any integer values of n for which the frequencies (C.4) become imaginary. Hence the appearance of an actual unstable mode requires that either the background is highly curved on the string scale and/or the p^+ component of the momentum is very large (which corresponds to a highly excited string state). This once again emphasises the stringy nature of this potential instability.

D. Penrose limit of $AdS_5 \times S^5$ in Poincaré coordinates

Although the Penrose limit of $AdS_5 \times S^5$ in Poincaré coordinates has already been discussed in ref. [22], it is worth reviewing the analysis here: firstly, we use different coordinates on the five-sphere, which leads initially to a "mixed" Rosen-Brinkman form of the maximally supersymmetric pp-wave; and secondly, it will be useful to compare this simple case with the more complicated geometry to follow. The metric on $AdS_5 \times S^5$, with the AdS factor in global coordinates, is

$$\mathrm{d}s^2 = L^2 \left[-\cosh^2\rho\, \mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \sinh^2\rho\, \mathrm{d}\Omega_3^2 + \cos^2\theta\, \mathrm{d}\phi^2 + \mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\hat{\Omega}_3^2 \right], \qquad \text{(D.1)}$$

where $\mathrm{d}\Omega_3^2$ and $\mathrm{d}\hat{\Omega}_3^2$ denote metrics on a unit three-sphere. In these coordinates, a simple class of null geodesics is that for which $\rho = 0 = \theta$. Taking the Penrose limit along such a geodesic which has angular momentum in the ϕ direction gives rise [20, 22, 21] to the maximally supersymmetric pp-wave of type-IIB supergravity [18]. Poincaré coordinates on AdS_5 are defined by

$$\begin{aligned} y &= \frac{1}{L}\left(\cosh\rho\cos\tau - \sinh\rho\,\Omega_4\right), \\ t &= \frac{1}{y}\cosh\rho\,\sin\tau, \\ x^i &= \frac{1}{y}\sinh\rho\,\Omega_i, \end{aligned} \qquad \text{(D.2)}$$

where $x^i, i = 1,2,3$, are the coordinates on $\mathbb{E}^3$ and where $\Omega_i\Omega_i + \Omega_4\Omega_4 = 1$ gives an embedding of S^3 in $\mathbb{E}^4$. Defining a new radial coordinate

$$r = L\ln(Ly), \qquad \text{(D.3)}$$

the metric on $AdS_5 \times S^5$ is thus

$$\mathrm{d}s^2 = e^{2r/L}\left[-\mathrm{d}t^2 + \mathrm{d}s^2(\mathbb{E}^3)\right] + \mathrm{d}r^2 + L^2\left[\cos^2\theta\ \mathrm{d}\phi^2 + \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\hat{\Omega}_3^2\right]. \qquad \text{(D.4)}$$

For geodesics at a constant point in $\mathbb{E}^3$, the effective lagrangian is

$$\mathcal{L} = -e^{2r/L}\dot{t}^2 + \dot{r}^2 + L^2\left[\cos^2\theta\dot{\phi}^2 + \dot{\theta}^2 + \sin^2\theta\dot{\hat{\Omega}}_3^2\right], \tag{D.5}$$

where, if λ is the affine parameter, a dot denotes $\mathrm{d}/\mathrm{d}\lambda$. One is still free to consider the class of geodesics for which $\theta = 0$. Then the t and ϕ equations give

$$\dot{t} = ELe^{-2r/L}, \qquad \dot{\phi} = h, \tag{D.6}$$

where E and h are the conserved energy and angular momentum associated with the Killing vectors $\partial/\partial t$ and $\partial/\partial\phi$, respectively. The null condition $\mathcal{L} = 0$ then gives

$$\dot{r} = \pm EL\sqrt{e^{-2r/L} - \frac{h^2}{E^2}}. \tag{D.7}$$

If we choose the $-$ sign in the above, then the resulting geodesic matches onto the $\rho = 0$ geodesics in global coordinates.[7] Integrating (D.7) and (D.6) gives

$$r(\lambda) = L\ln\left(\frac{E}{h}\cos\lambda h\right), \qquad t(\lambda) = L\frac{h}{E}\tan\lambda h. \tag{D.8}$$

Transforming back to the y coordinate defined in (D.2) above, gives

$$y(\lambda) = \frac{1}{L}\frac{E}{h}\cos\lambda h, \tag{D.9}$$

which, with $h = E$, matches onto the $\rho = 0$ null geodesics in global coordinates, as promised (and these latter do indeed have $h = E$).

Following ref. [22], we introduce coordinates $\{u, v, x\}$ such that u is the affine parameter along the null geodesics. Demanding that $g_{uu} = 0 = g_{ux}$ and $g_{uv} = 1$, a possible choice is

$$\begin{aligned}
\partial_u &= \dot{r}\partial_r + \dot{t}\partial_t + \dot{\phi}\partial_\phi, \\
&= -Lh\tan uh\,\partial_r + \frac{h^2L}{E}\sec^2 uh\,\partial_t + h\partial_\phi, \\
\partial_v &= -\frac{1}{EL}\partial_t, \\
\partial_x &= \frac{1}{L}\partial_\phi + \frac{h}{E}\partial_t,
\end{aligned} \tag{D.10}$$

which can be integrated to give

$$\begin{aligned}
r(u) &= L\ln\left(\frac{E}{h}\cos uh\right), \\
t(u,v,x) &= \frac{hL}{E}\tan uh - \frac{v}{EL} + \frac{h}{E}x, \\
\phi(u,x) &= \frac{x}{L} + hu.
\end{aligned} \tag{D.11}$$

[7] The plus sign gives rise to a second class of null geodesics, the Penrose limit along which has precisely the same effect on the spacetime, so we will not consider it here.

We now write the original metric (D.4) in terms of $\{u, v, x\}$ and implement the fact that $\theta = 0$ by defining $\theta = y/L$ and taking the limit $L \to \infty$. Dropping terms of $\mathcal{O}(1/L)$ gives the metric

$$\mathrm{d}s^2 = 2\mathrm{d}u\mathrm{d}v - h^2y^2\mathrm{d}u^2 + \sin^2 uh\, \mathrm{d}x^2 + \frac{E^2}{h^2}\cos^2 uh \mathrm{d}s^2\left(\mathbb{E}^3\right) + \mathrm{d}s^2\left(\mathbb{E}^4\right), \tag{D.12}$$

where y is the radial coordinate on $\mathbb{E}^4$. The coordinate singularities in this metric appear because of degeneracies in the choice of vectors in C.4. For example, $\partial_u = hL\partial_x$ at $\sin uh = 0$. Further, we note in passing that working in global coordinates, and the analogue thereof on the sphere, gives rise to the pp-wave in Brinkman coordinates. Use of Poincaré coordinates, however, and the parametrisation of the five-sphere used in ref. [22], gives rise to the pp-wave in Rosen coordinates. The "mixed" coordinates used here has given rise to the above pp-wave in "mixed" Brinkman-Rosen coordinates. At any rate, introducing

$$\begin{aligned} x^- &= u\,, \qquad z = \sin uhx\,, \qquad z^i = \frac{E}{h}\cos uhx^i\,, \\ x^+ &= v + \frac{1}{4}\left(\frac{E}{h}x^ix^i - x^2\right)\sin(2uh)\,, \end{aligned} \tag{D.13}$$

gives

$$\mathrm{d}s^2 = 2\mathrm{d}x^-\mathrm{d}x^+ - h^2|x|^2\mathrm{d}x^{-2} + \mathrm{d}s^2\left(\mathbb{E}^8\right), \tag{D.14}$$

where now $|x|$ denotes the radial coordinate on $\mathbb{E}^8$.

As to the R-R five-form field strength which, in Poincaré coordinates, has the form

$$F_5 = \frac{C}{L}(1+\star)\epsilon(AdS_5) = \frac{C}{L}e^{4r/L}(1+\star)\mathrm{d}t \wedge \epsilon\left(\mathbb{E}^3\right) \wedge \mathrm{d}r\,, \tag{D.15}$$

for some constant C and where $\epsilon(\mathcal{M})$ denotes the volume form on $\mathcal{M}$, we find

$$F_5 = C\mathrm{d}x^- \wedge \epsilon\left(\mathbb{E}^4\right). \tag{D.16}$$

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